Foundations and Applications of Zero-Knowledge Proofs

4 Sept 2024

zkSNARKs for Blockchains

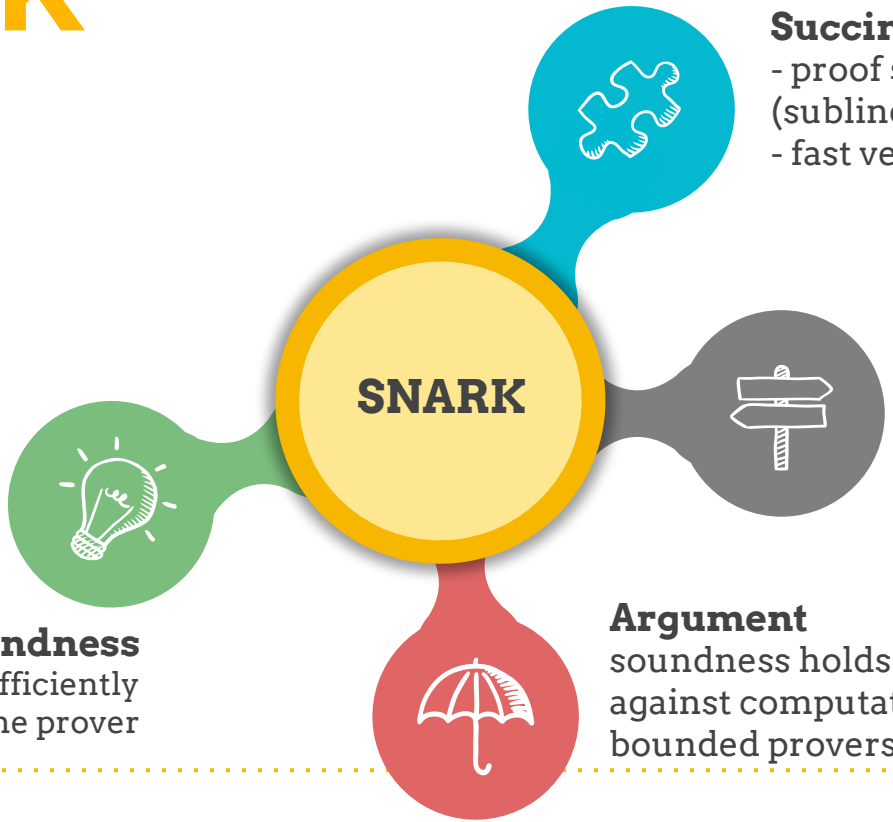
Anca Nitulescu

INPUT | OUTPUT

SNARKs

Background

SNARK



Succinctness
- proof size independent (sublinear) of NP witness size
- fast verification

Non-Interactivity
no exchange between prover and verifier

Argument
soundness holds only against computationally bounded provers

Knowledge Soundness
a witness can be efficiently extracted from the prover

zk-SNARK

Zero-Knowledge
does not leak anything about the witness

Succinctness
- proof size independent (sublinear) of NP witness size
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Knowledge Soundness
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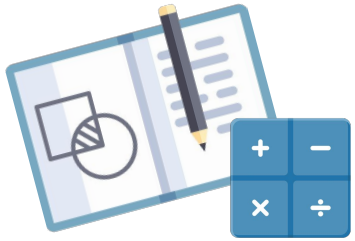
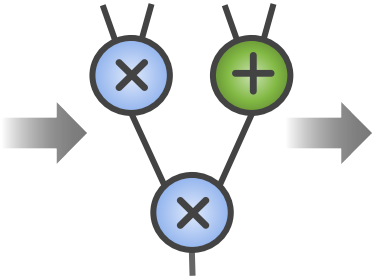
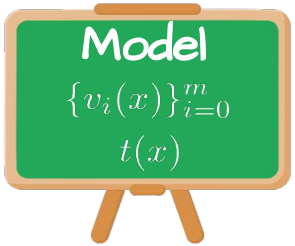
Building SNARKs







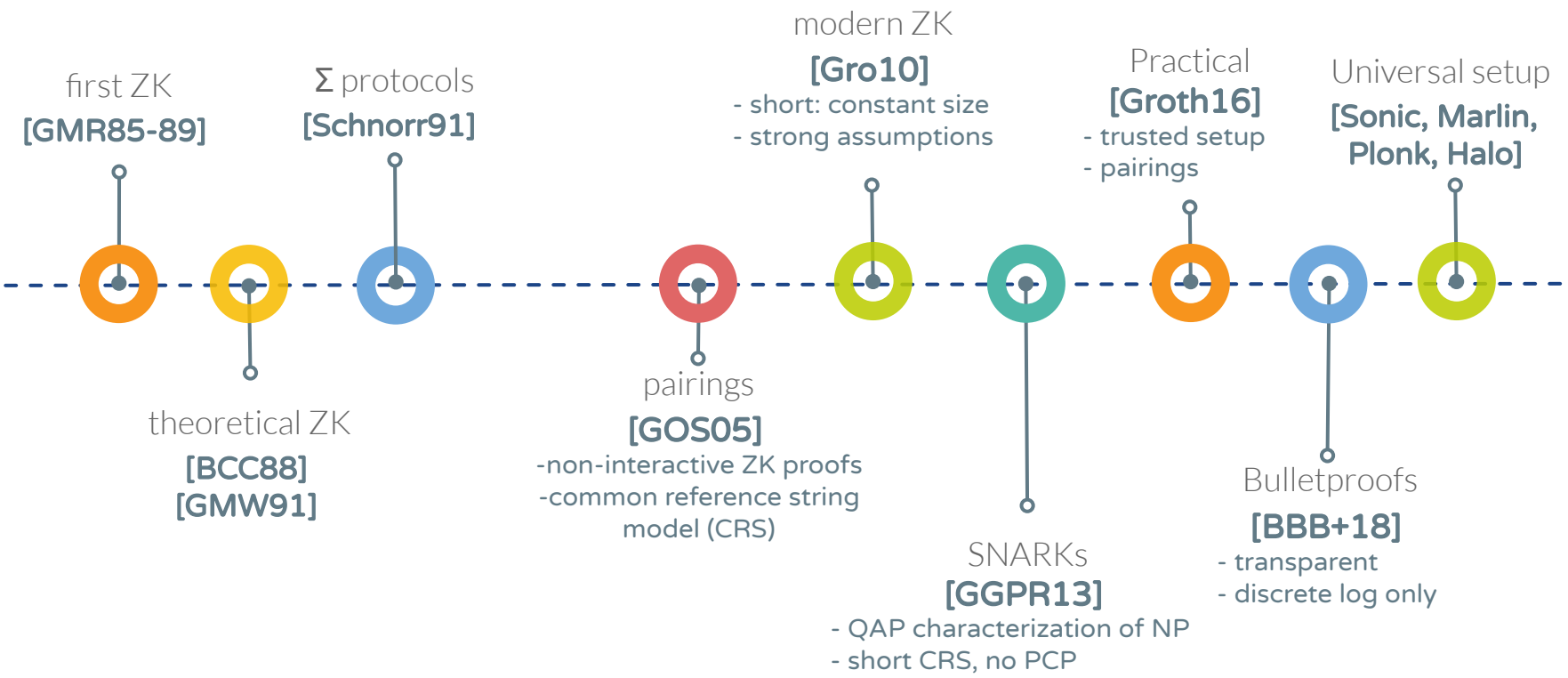
Target Statement
∠,R(y,w)=1

Circuit SAT
NP-complete

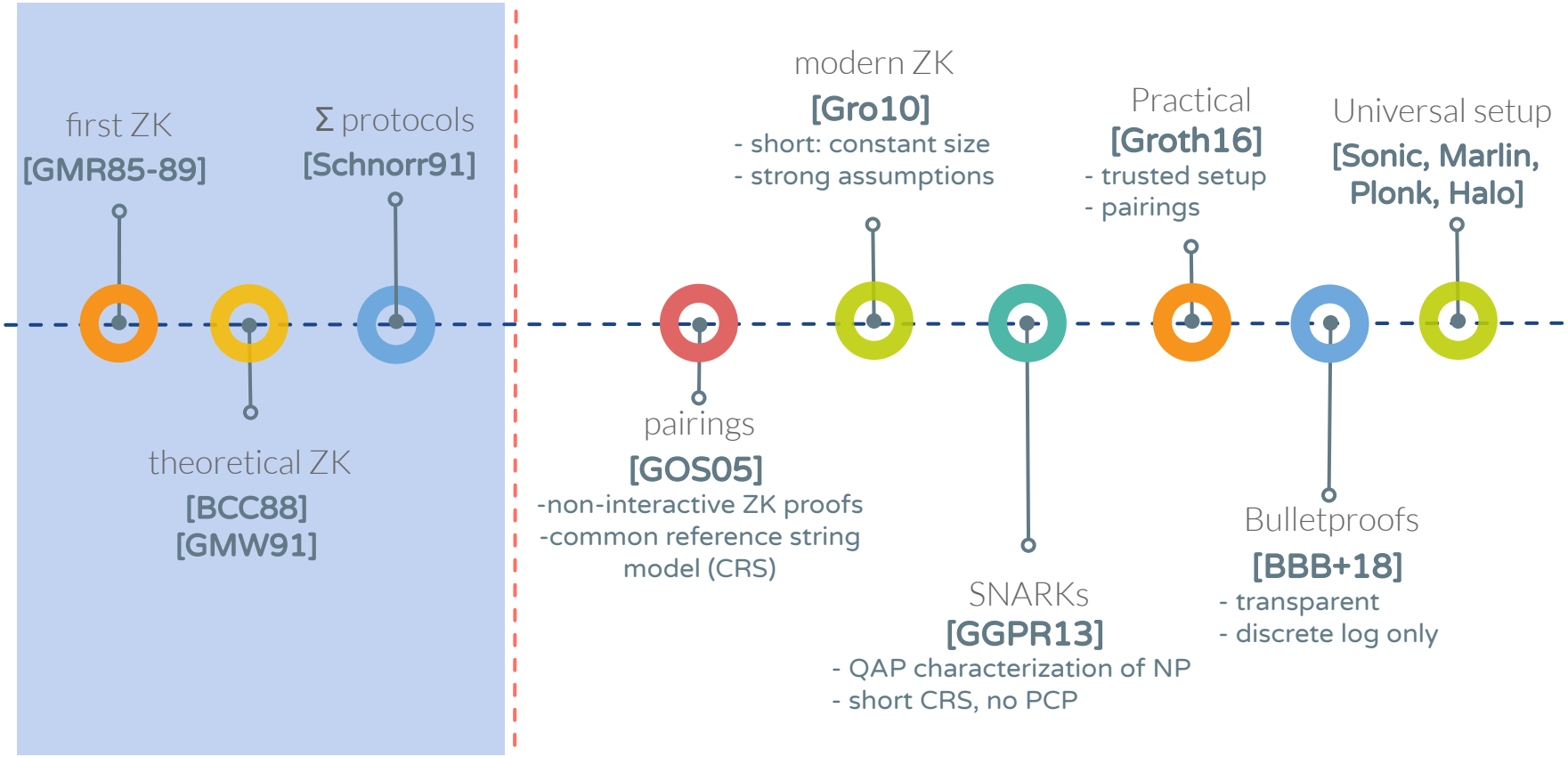
Computational Model
(Polynomials)

Cryptographic Compilation

Non-Interactive ZK Proofs



Non-Interactive ZK Proofs



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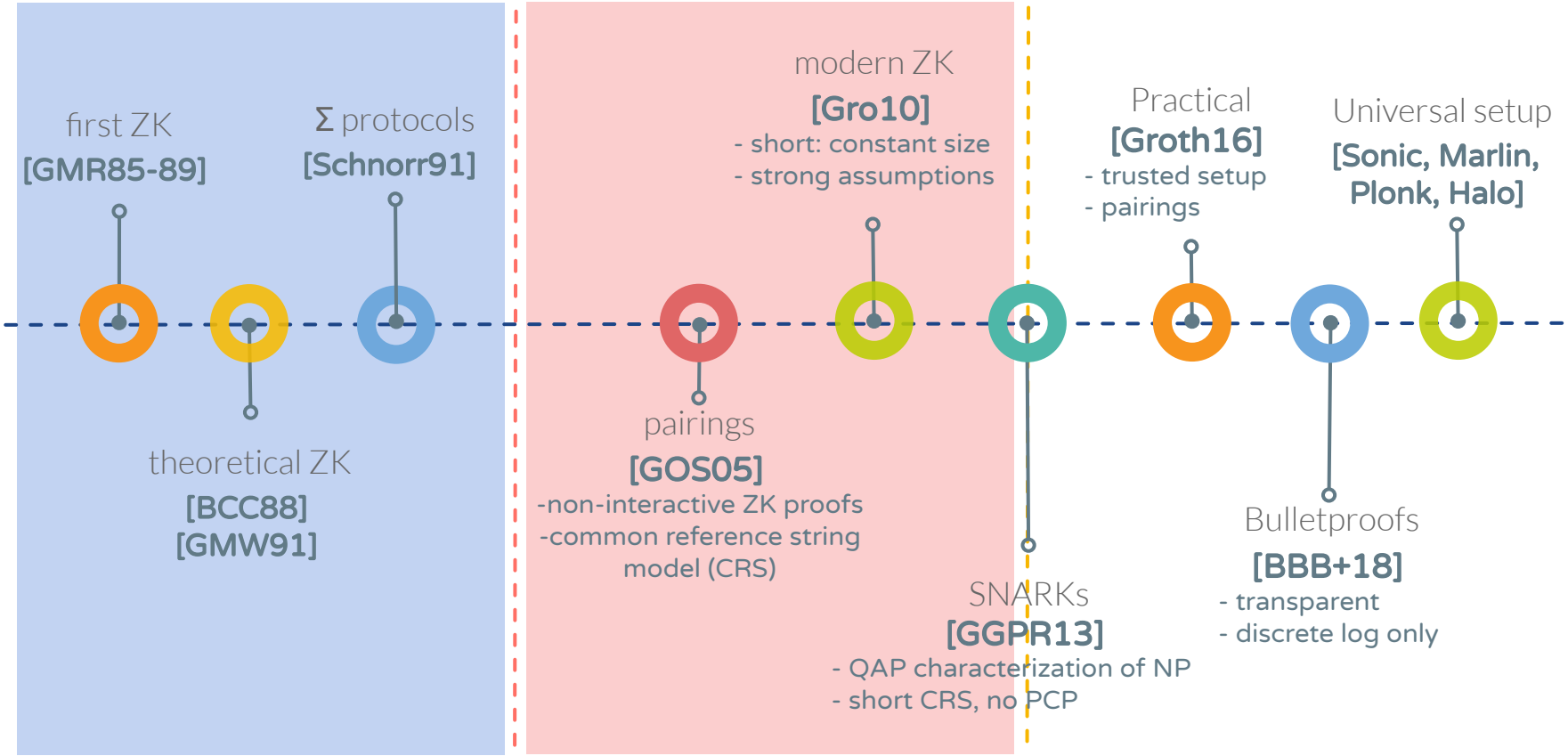
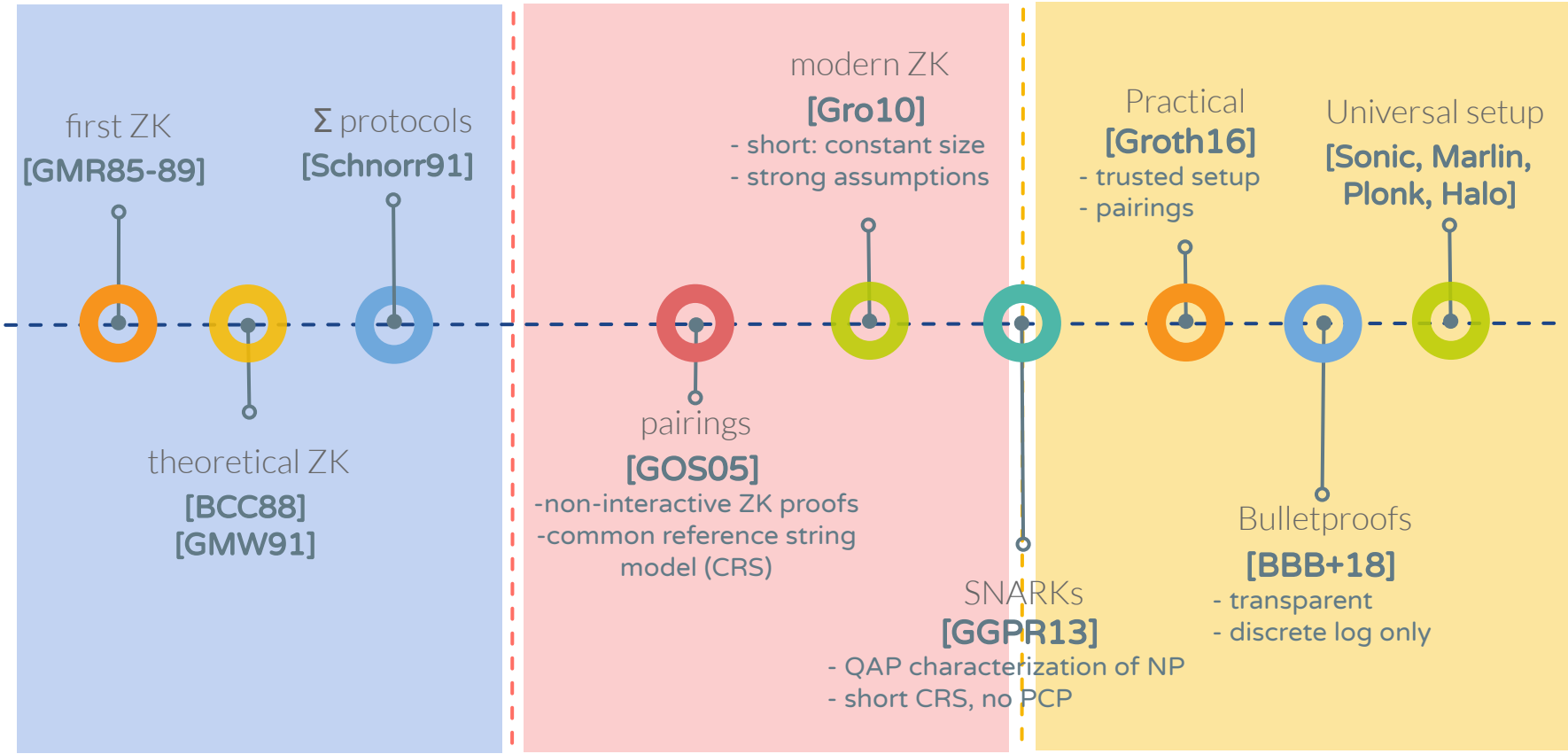
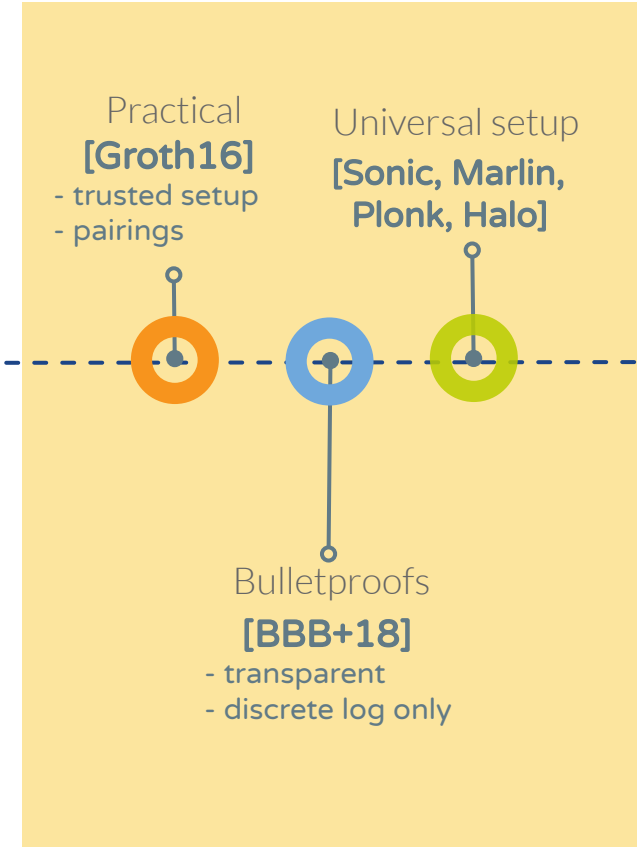


Non-Interactive ZK Proofs



Recent zk-SNARK research



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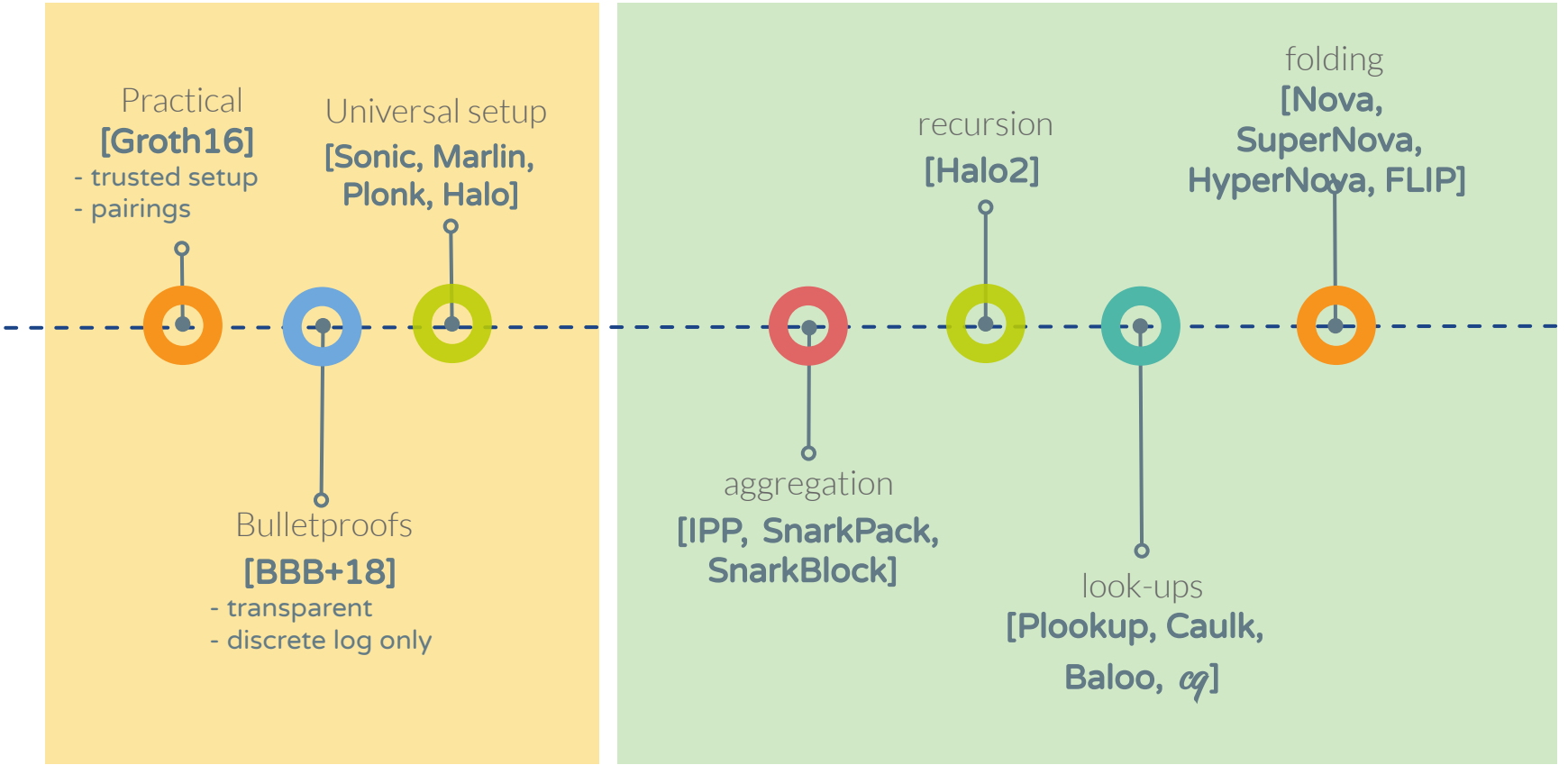
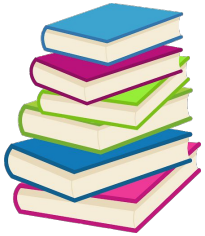


References



[GMR85-89] **The Knowledge Complexity Of Interactive Proof Systems,** by Shafi Goldwasser, Silvio Micali, and Charles Rackoff

[GMW91] **How to prove all NP-statements in zero-knowledge, and a methodology of cryptographic protocol design,** by Oded Goldreich, Silvio Micali, and Avi Wigderson

[BCC88] **Minimum Disclosure Proofs of Knowledge**, by Gilles Brassard, David Chaum, and Claude Crépeau

[Sch91] **Efficient signature generation by smart cards,** by Claus-Peter Schnorr, Journal of Cryptology, 1991.

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[Groth10] **Short pairing-based non-interactive zero-knowledge arguments**, Jens Groth, ASIACRYPT 2010.

[KZG10] **Constant-Size Commitments to Polynomials and Their Applications**, by A. Kate, G. Zaverucha, I. Goldberg, in ASIACRYPT'10.

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[BBB+18] **Bulletproofs: Short Proofs for Confidential Transactions and More**, by B. Bünz, J. Bootle, D. Boneh, A. Poelstra, P. Wuille, and G. Maxwel

[Groth16] **On the Size of Pairing-based Non-interactive Arguments**, Jens Groth, EUROCRYPT 2016.

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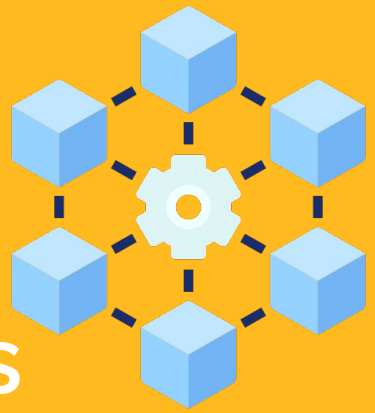
Applications

to descentralized systems

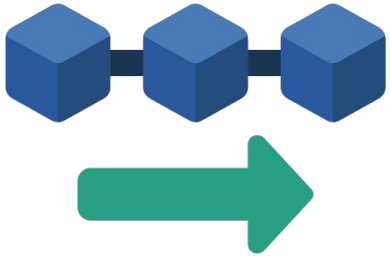


Crypto currencies

Distributed Ledger

Alice:	20 Coins
Bob:	55 Coins
Carol:	0 Coins
...	
Zoe:	17 Coins

tx: Alice → Bob, **5 Coins**



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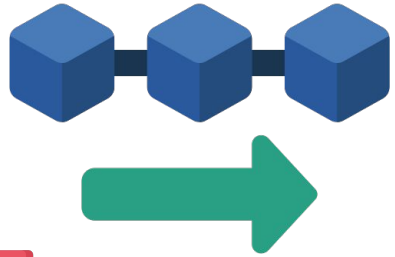

New State

Alice:	15 Coins
Bob:	60 Coins
Carol:	0 Coins
...	
Zoe:	17 Coins

Crypto currencies

Distributed Ledger

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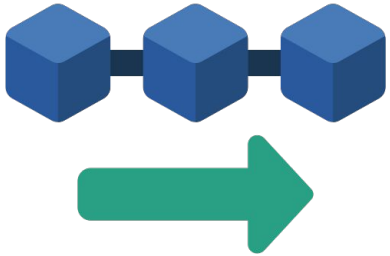
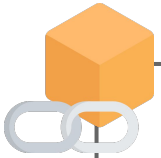


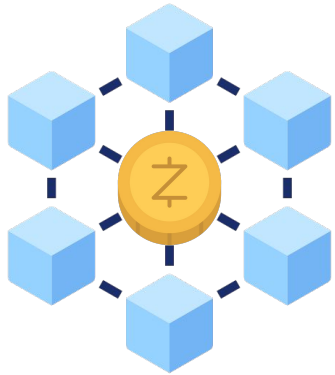


- inclusive accountability:
  anyone can see and check validity of actions
- all transactions are public
- consensus: multiple nodes check a new block

ZK Proofs **in Distributed Systems**

Privacy:

- Cryptocurrencies that guarantee privacy TX on public Ledger
- **Zero Knowledge** Proof: the TX is valid without revealing more
    - ZCash, Aleo, Aztec, Midnight



Compliance:

- Prove that a private TX is compliant with banking laws
    - Esspresso Systems
- Prove that an exchange is solvent
  e.g. a company has more assets than obligations to its customers
- **Zero-Knowledge** to protect customers private data

ZK **Proofs in Distributed Systems**

Distributed storage:

- Peer-to-peer network for storage
- Allows anyone to store and retrieve data on the internet
- built-in economic incentives ensure that files are stored reliably and continuously
- **Proof of Knowledge** of the stored file
    - Filecoin

ZK **Proofs in Distributed Systems**

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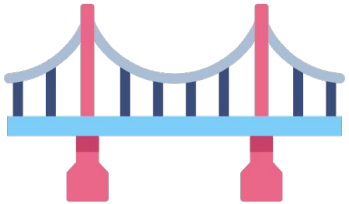
Bridging Blockchains:

- Transfer of assets from a chain to another
- Source chain proves to target chain that the asset is locked-up
- No need for Zero Knowledge
- Need for **Proof of Knowledge**



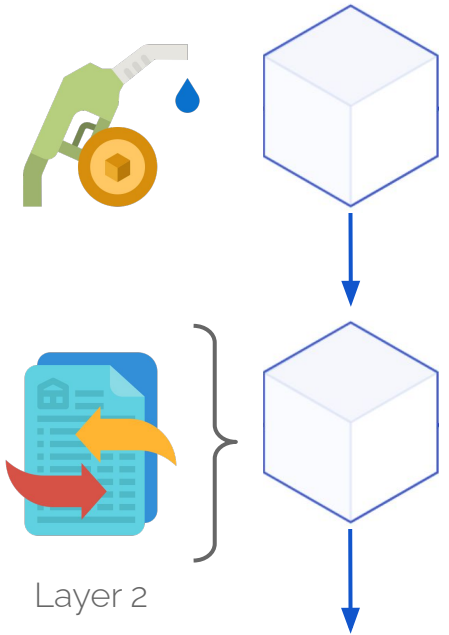
ZK **Proofs in Distributed Systems**

Rollups = Outsourced Computation

- Scalability tool
- Off-chain services (Layer 2) process a batch of TX (validates TX) and proves the work
- Main chain (Layer 1) verifies efficiently the work of an off-chain service
- Reduces the cost of transactions

- **No need** for Zero Knowledge
- Need for **Succinctness & Proof of Knowledge**

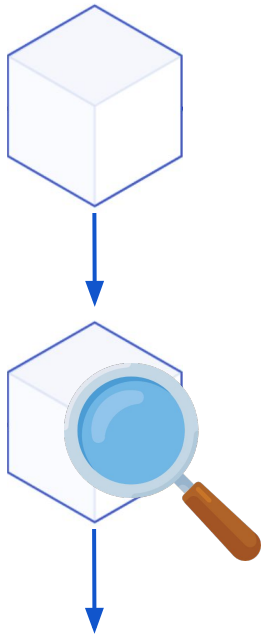
No zk: ZKSync, Loopring, Linea by Consensys, ZKSpace
With zk: Aztec, Polygon Hermez



ZK **Proofs Key Requirements**

Key Properties for Proofs in Blockchains

- **non-interactivity**
- publicly verifiable
- **succinctness for proofs**
- **efficiency for verifier**



Answer: SNARKs

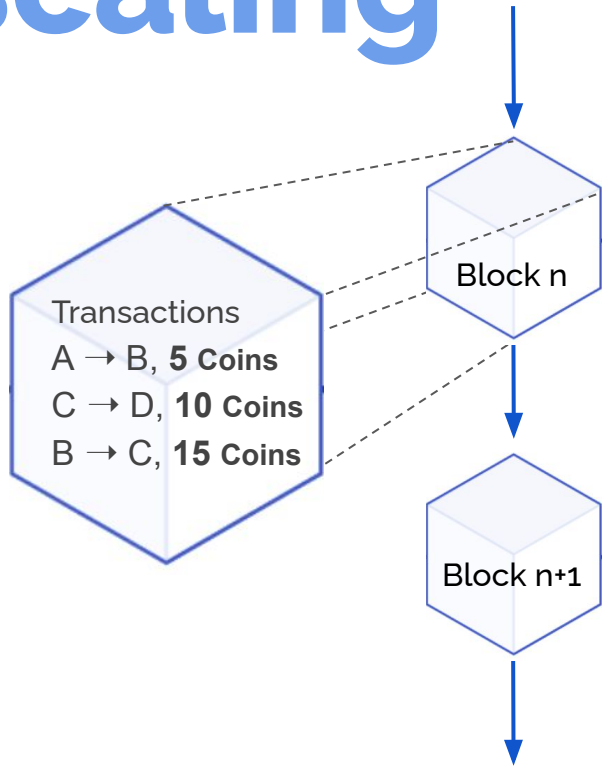
Succinct Non-Interactive Arguments of Knowledge

What is left?

Blockchain scaling

ZK Roll-ups

- instead of posting all transaction data on-chain
- execute **tx** using off-chain computation
- a summary of the state changes is published to the chain and verified



Blockchain scaling

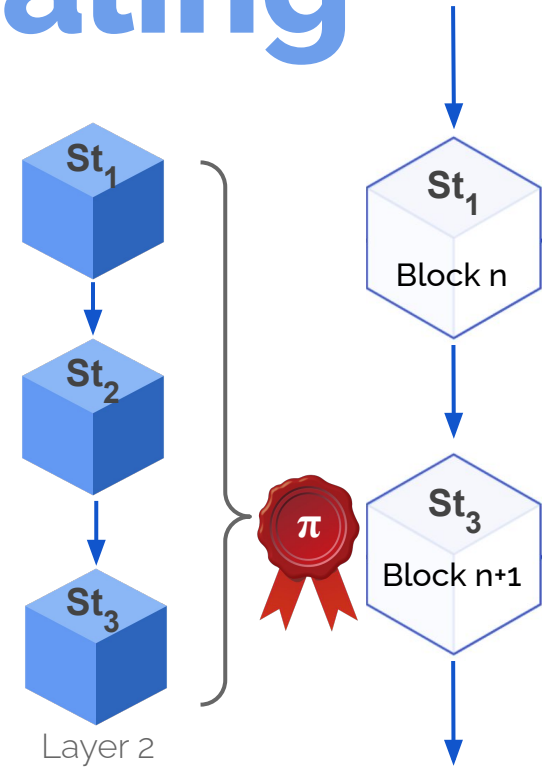
ZK Roll-ups

- instead of posting all transaction data on-chain
- execute **tx** using off-chain computation
- a summary of the state changes is published to the chain and verified

SNARK π proves the correctness of the changes

Statement: state changes proposed by the layer 2 are correct,

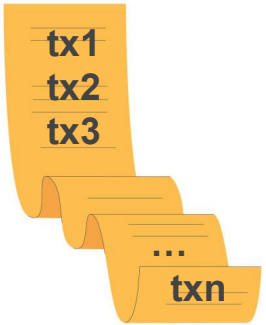
i.e. are the result of the execution of the given batch of transactions

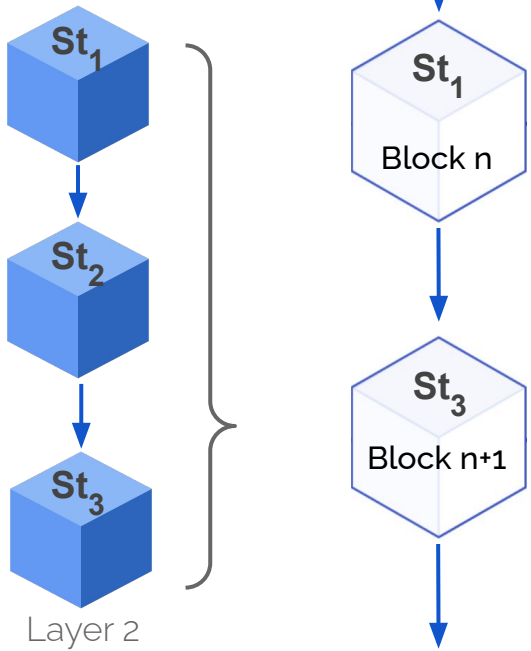


Blockchain scaling

ZK Roll-ups

Prove multiple instances:
- correctness of the execution of batch of **tx**



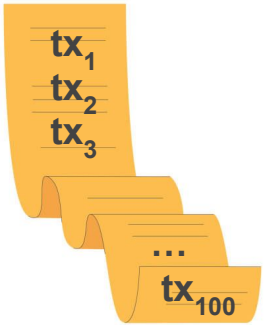


Blockchain scaling

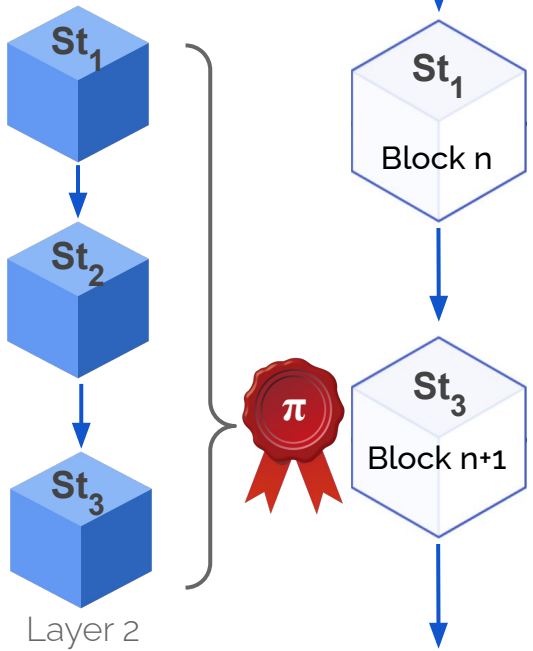
ZK Roll-ups

Example 100 **Tx**

Naively: generate state transition proof once all 100 Tx are submitted → **long delay**



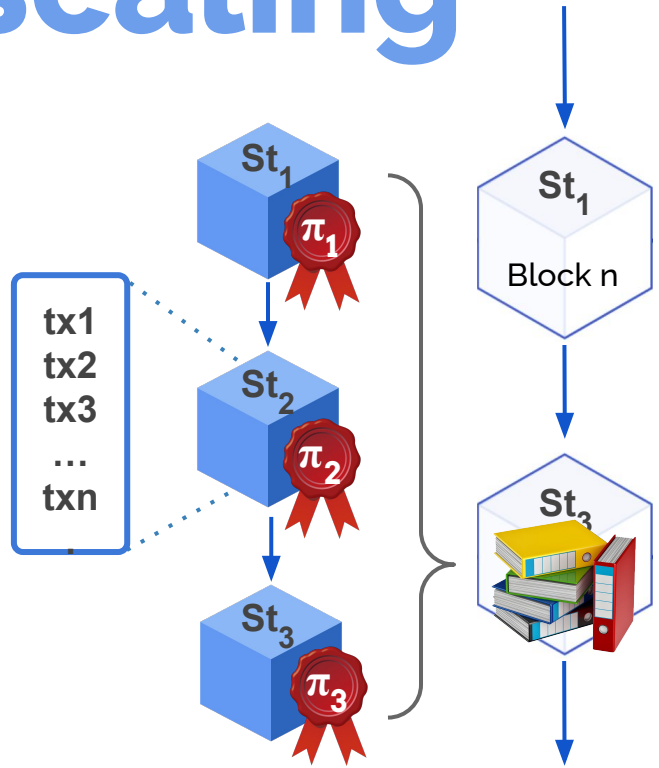
Proving is slow



Blockchain scaling

ZK Roll-ups

Periodically provide proofs for valid transaction batches



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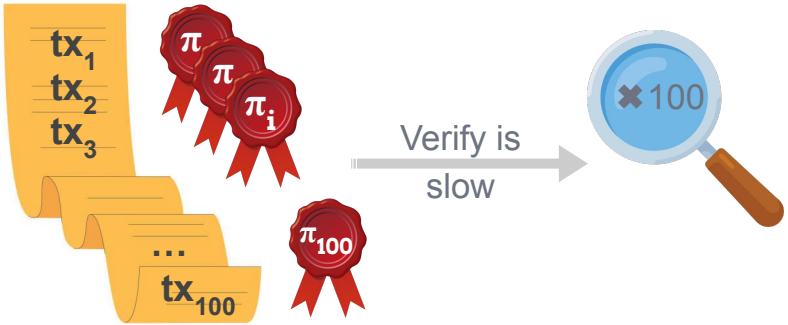
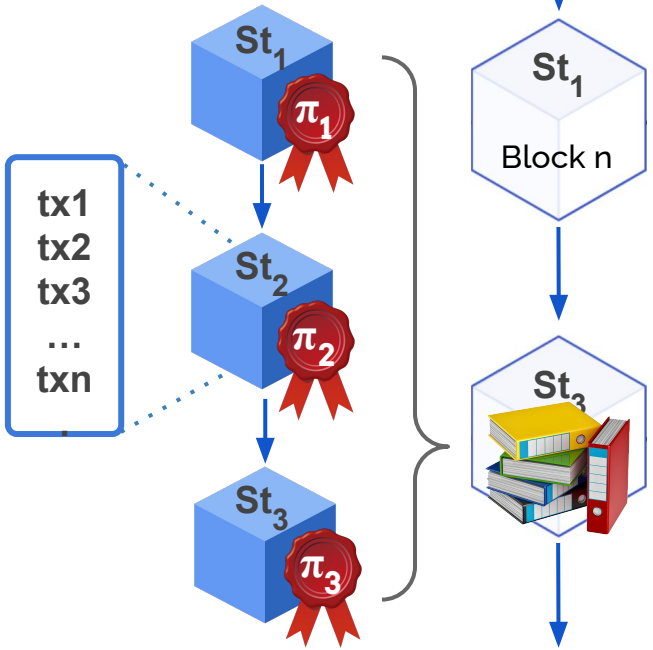
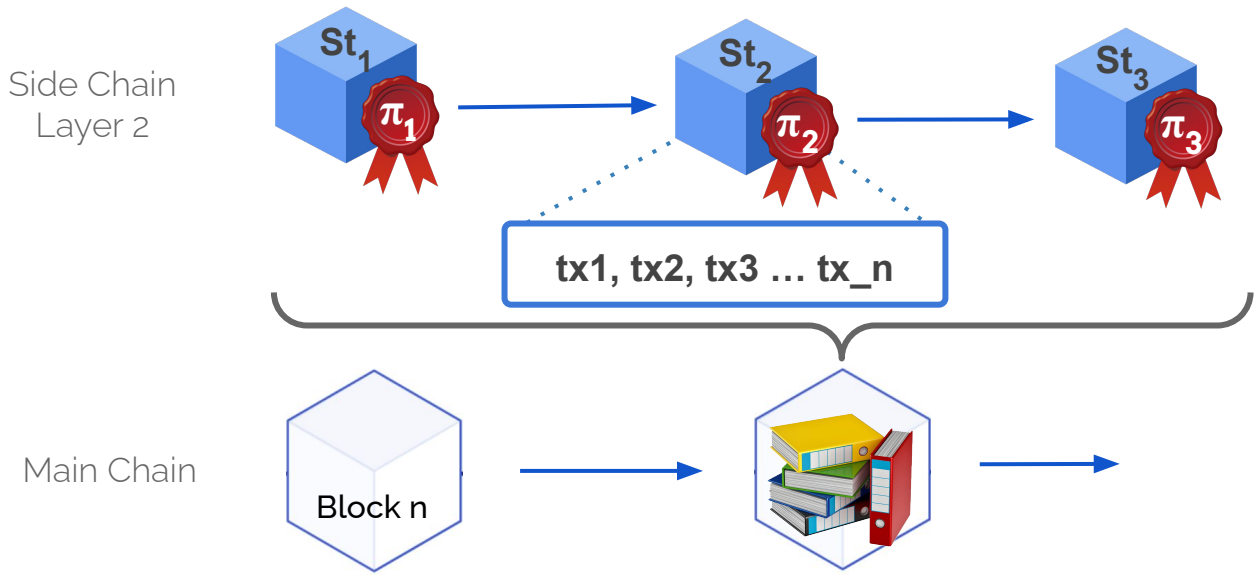
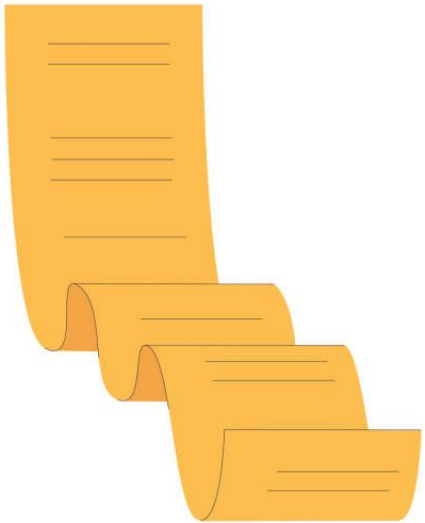




Prove & scale ?

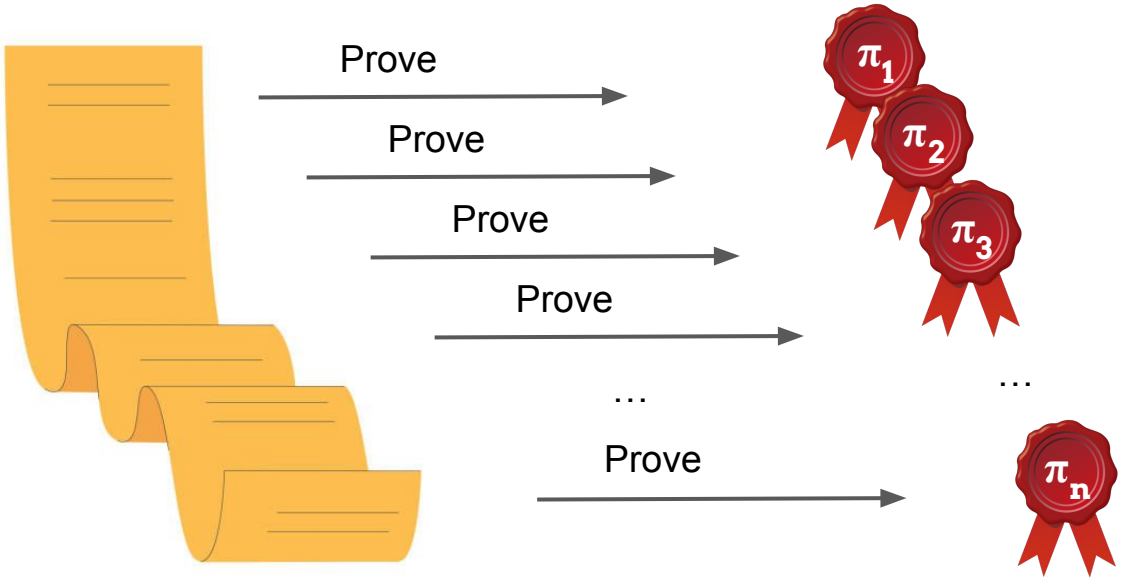

Side Chain
Layer 2
St1
π1
St2
π2
St3
π3
tx1, tx2, tx3 … tx_n
Main Chain
Block n

In Brief



Multiple instances to prove

Naively



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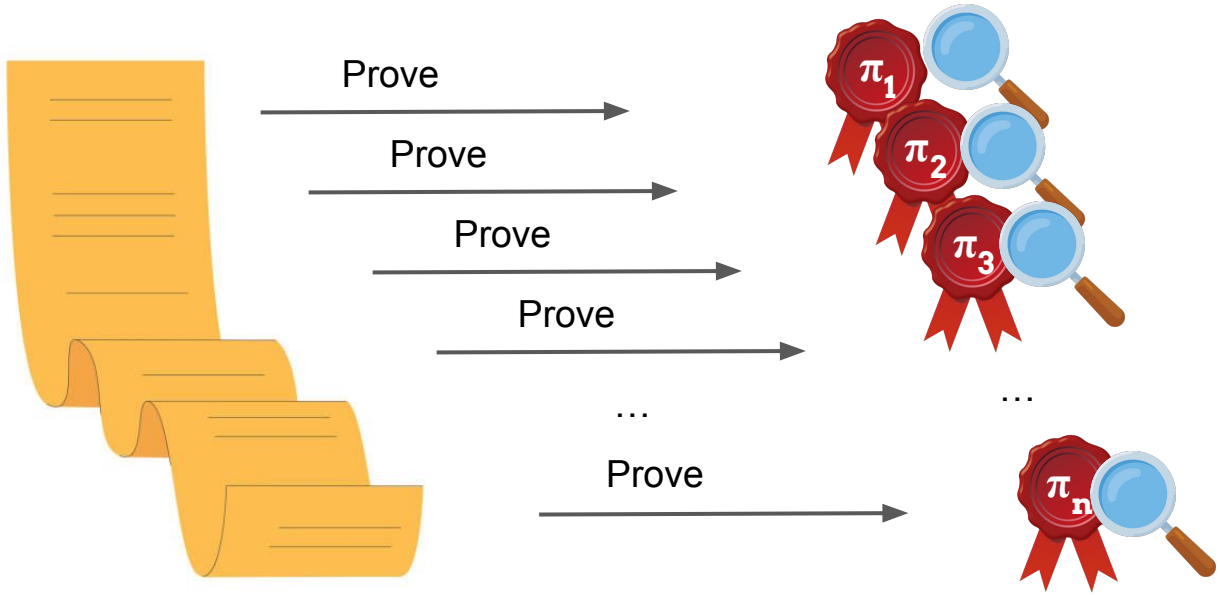
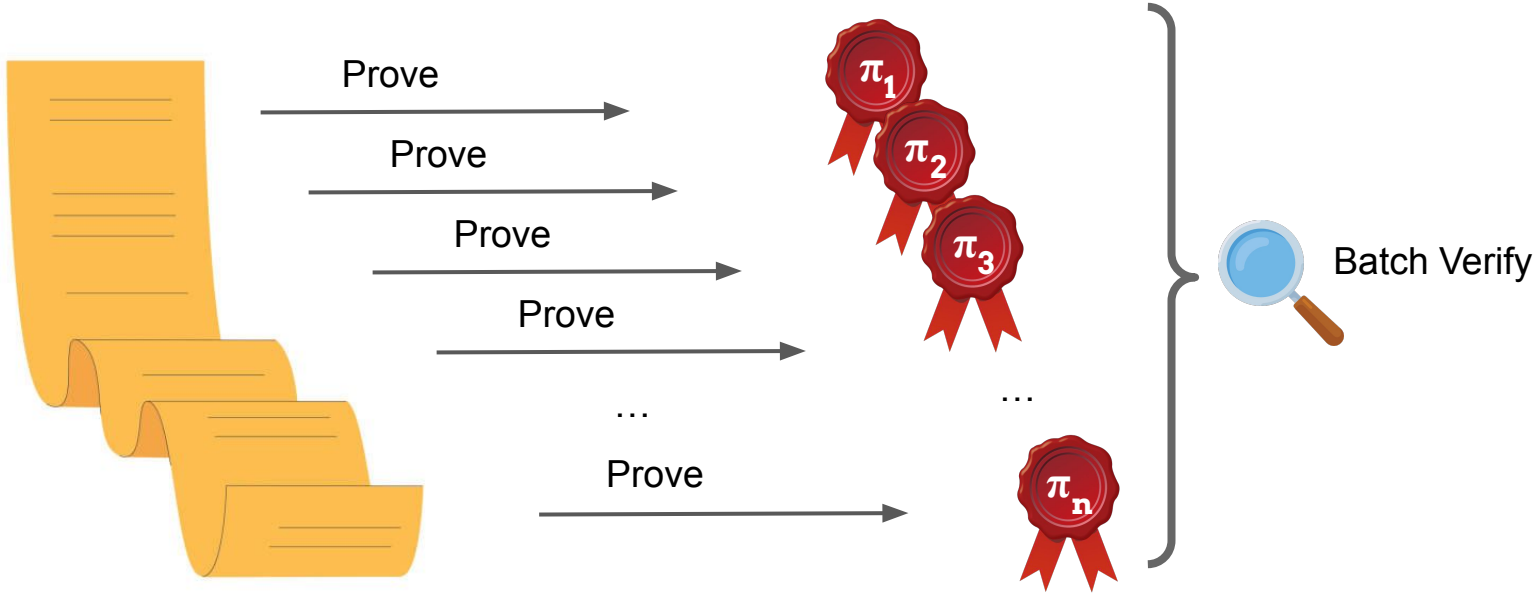


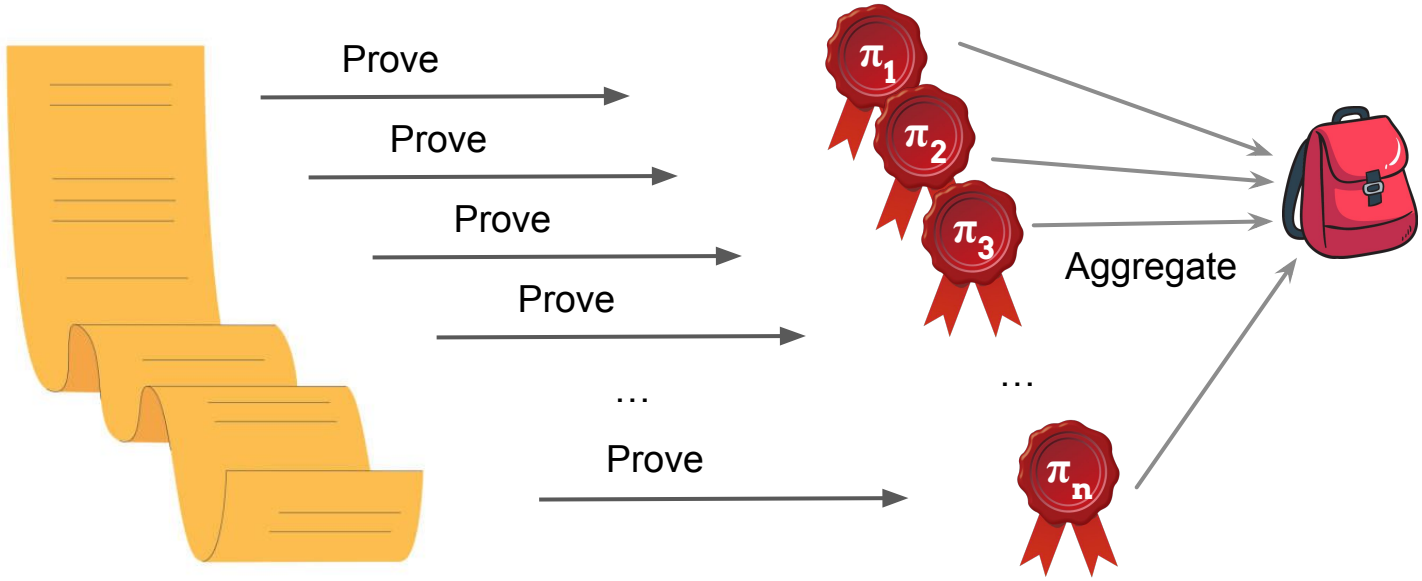
Multiple instances to prove

Batch Verify



Multiple instances to prove

Aggregation



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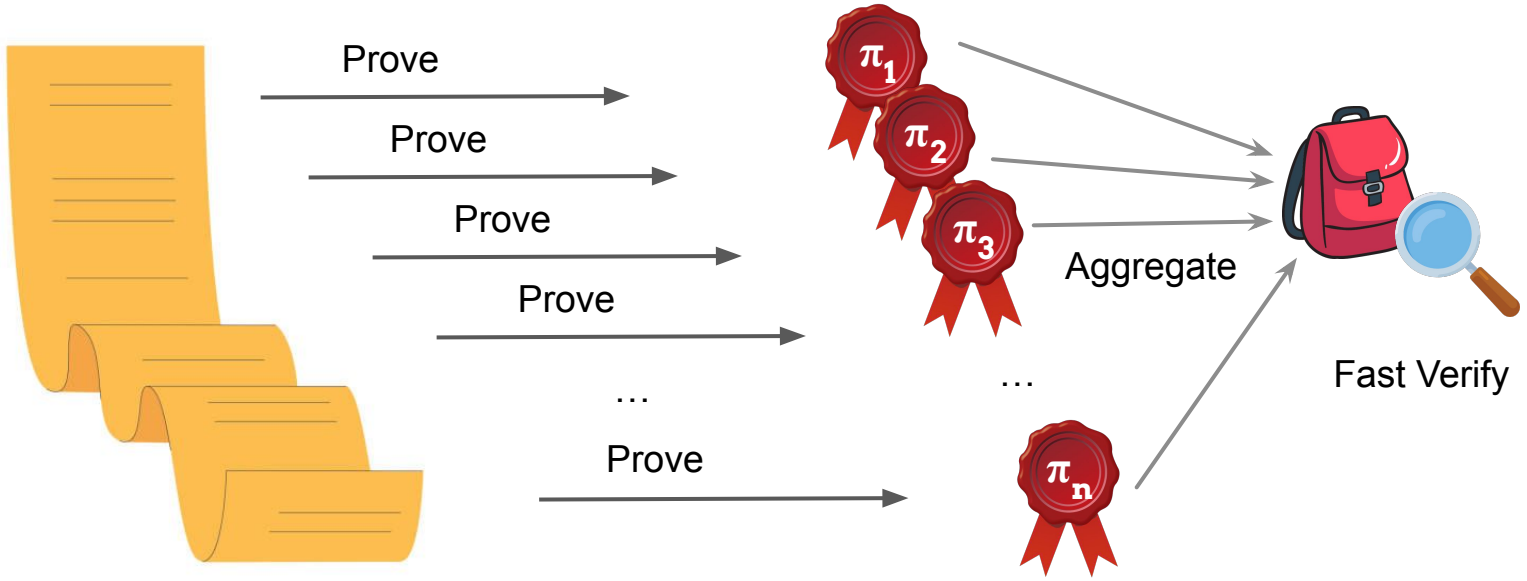
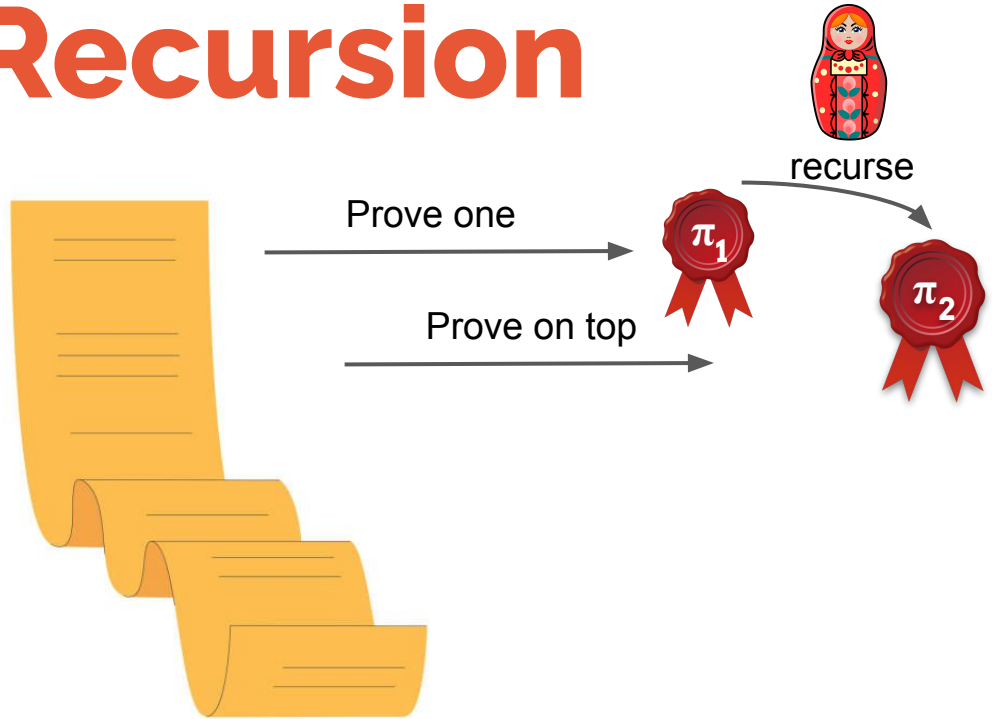


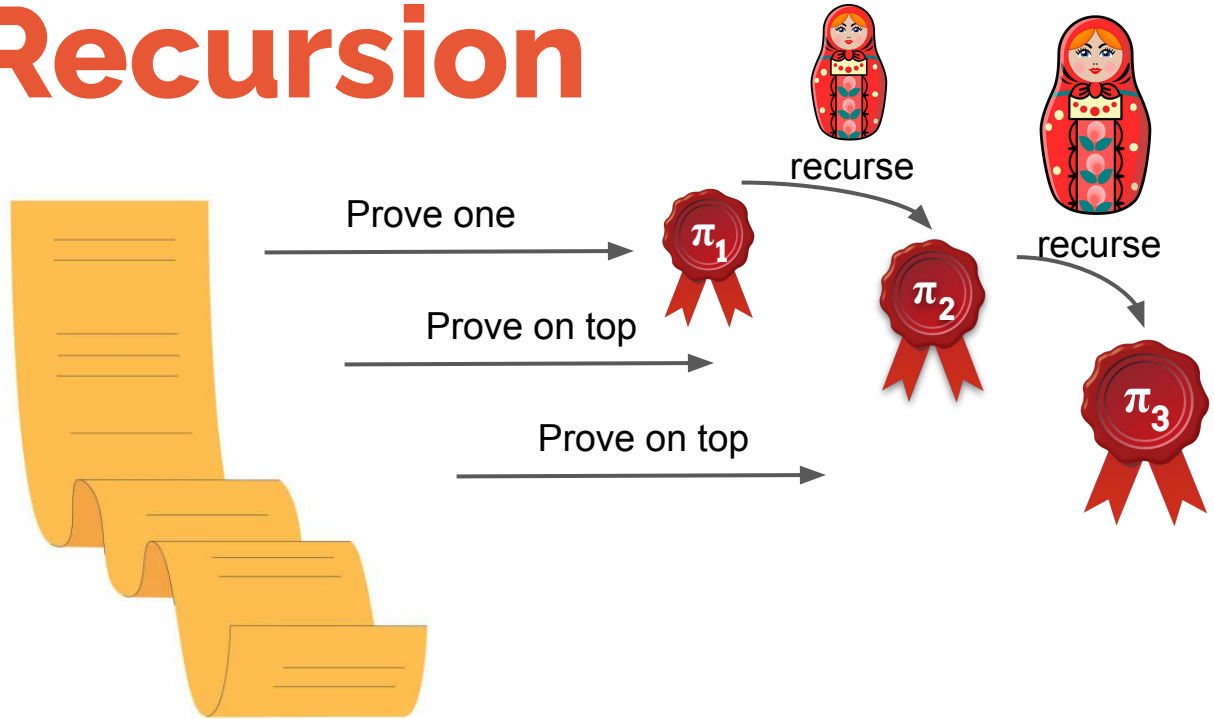
Multiple instances to prove

Recursion



Multiple instances to prove

Recursion



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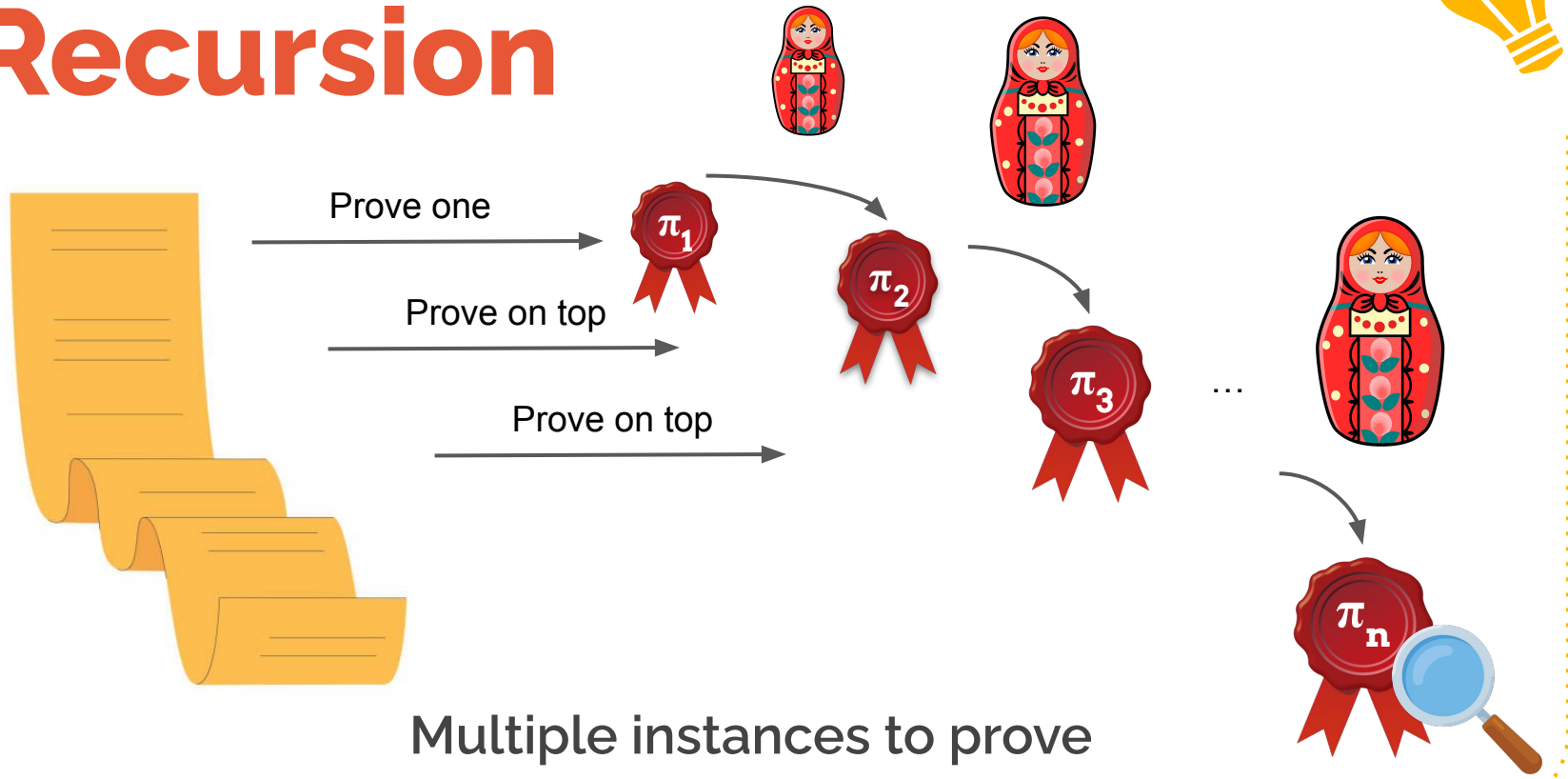
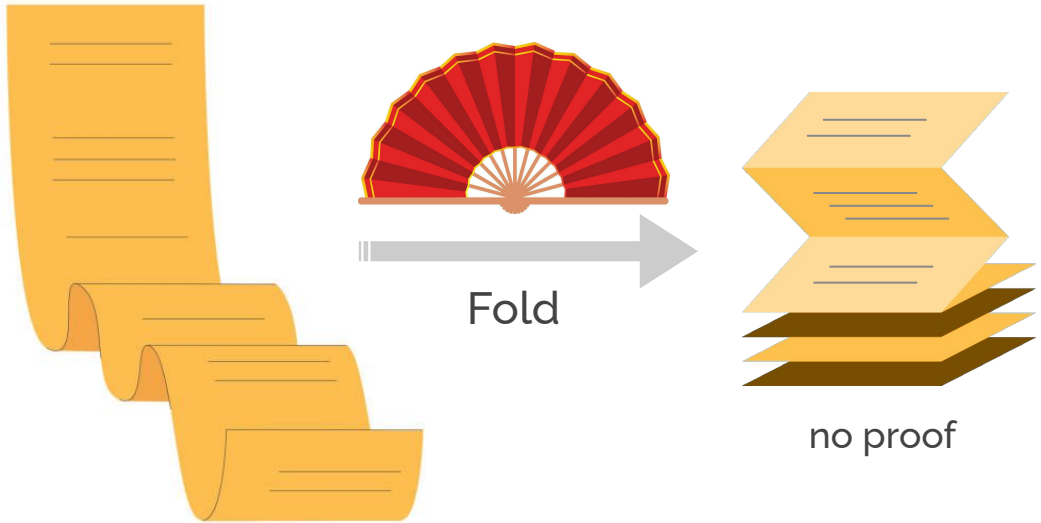


Multiple instances to prove

Folding



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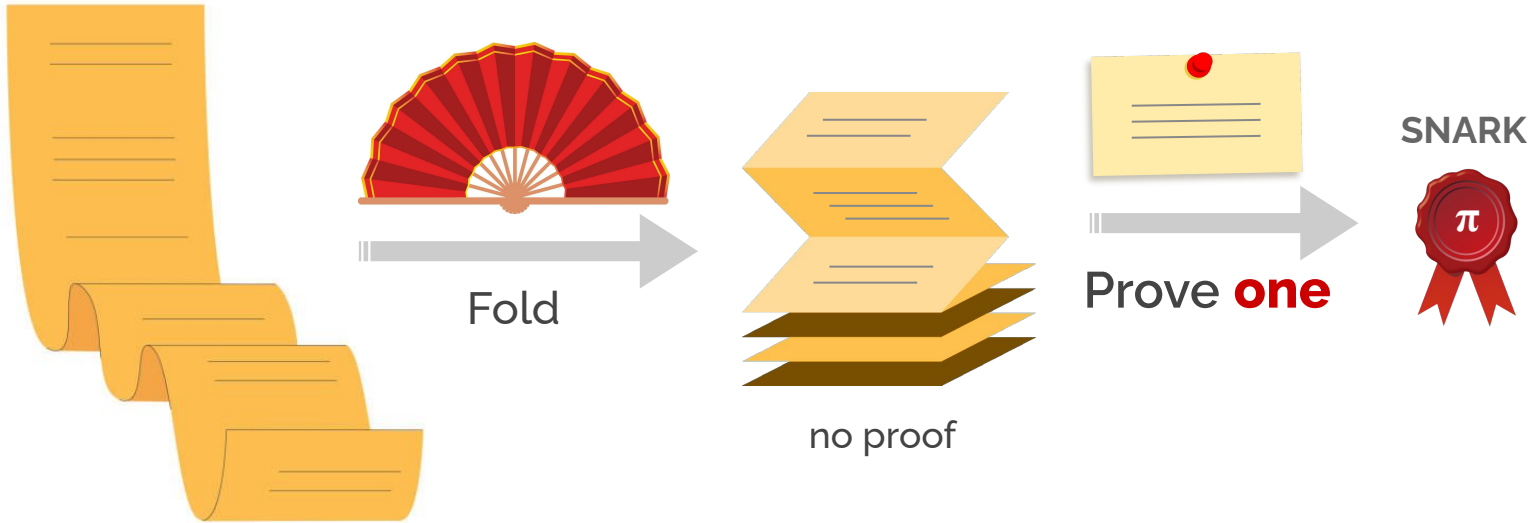

Multiple instances to prove

Prove many instances

Aggregation

Recursion

Folding

Proof Size

Verifier Time

Prover Time

SNARK Aggregation

Prove many instances

Aggregation

Proof Size

Verifier Time

Prover Time

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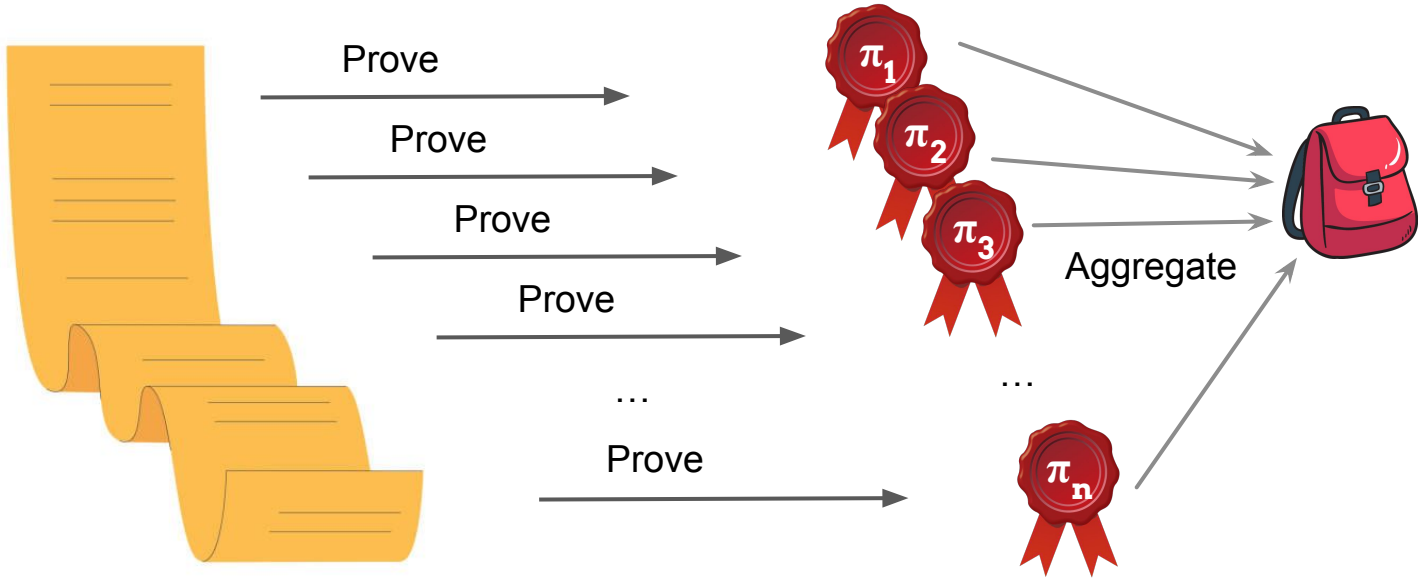


Multiple instances to prove

Distributed storage

Storage Providers

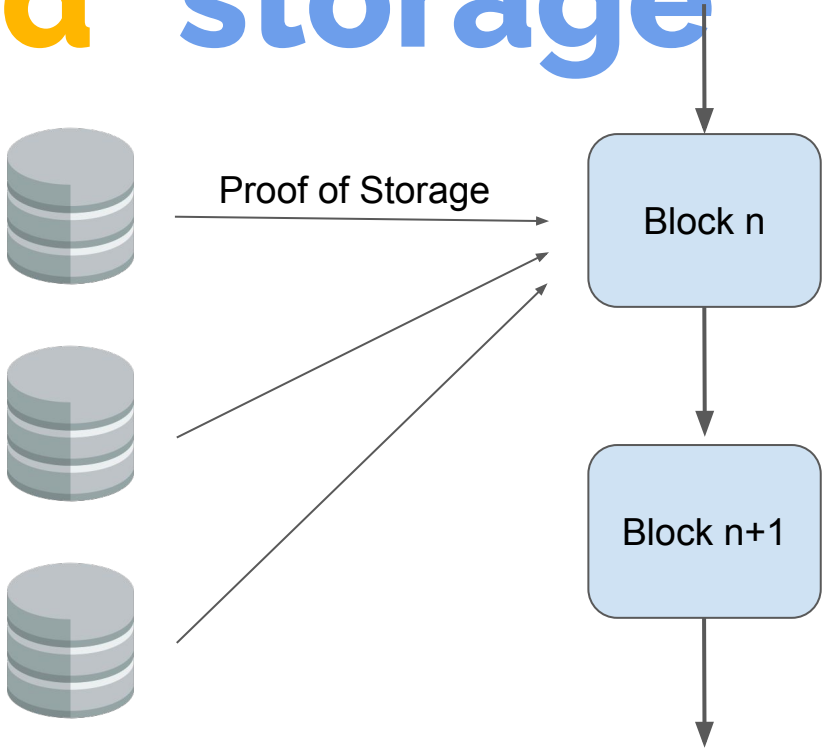
- onboard storage capacity
- earn block rewards
- regularly prove the storage

= **Provers**

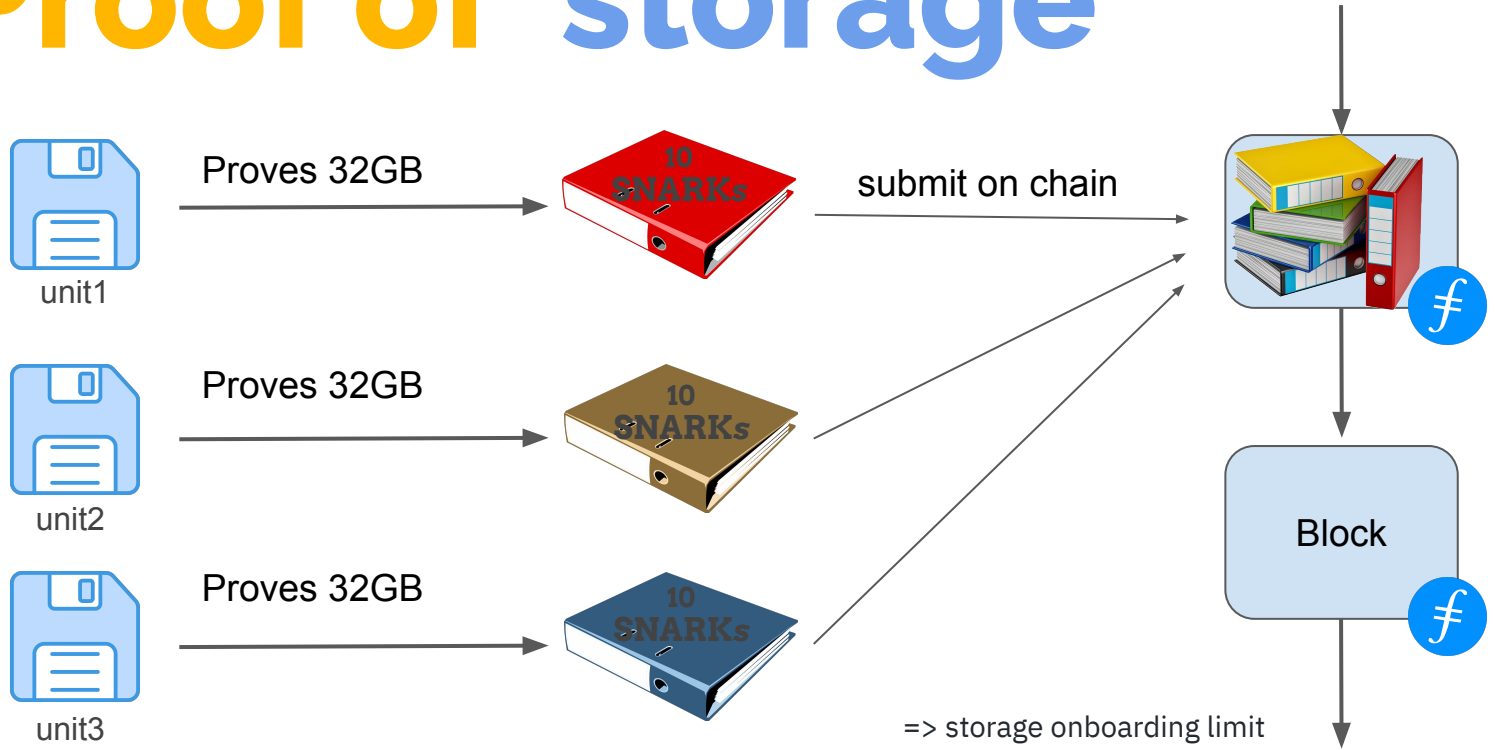
Nodes in network

- ensure data is being stored, maintained, and secured
- need to check proofs of space

= **Verifiers**



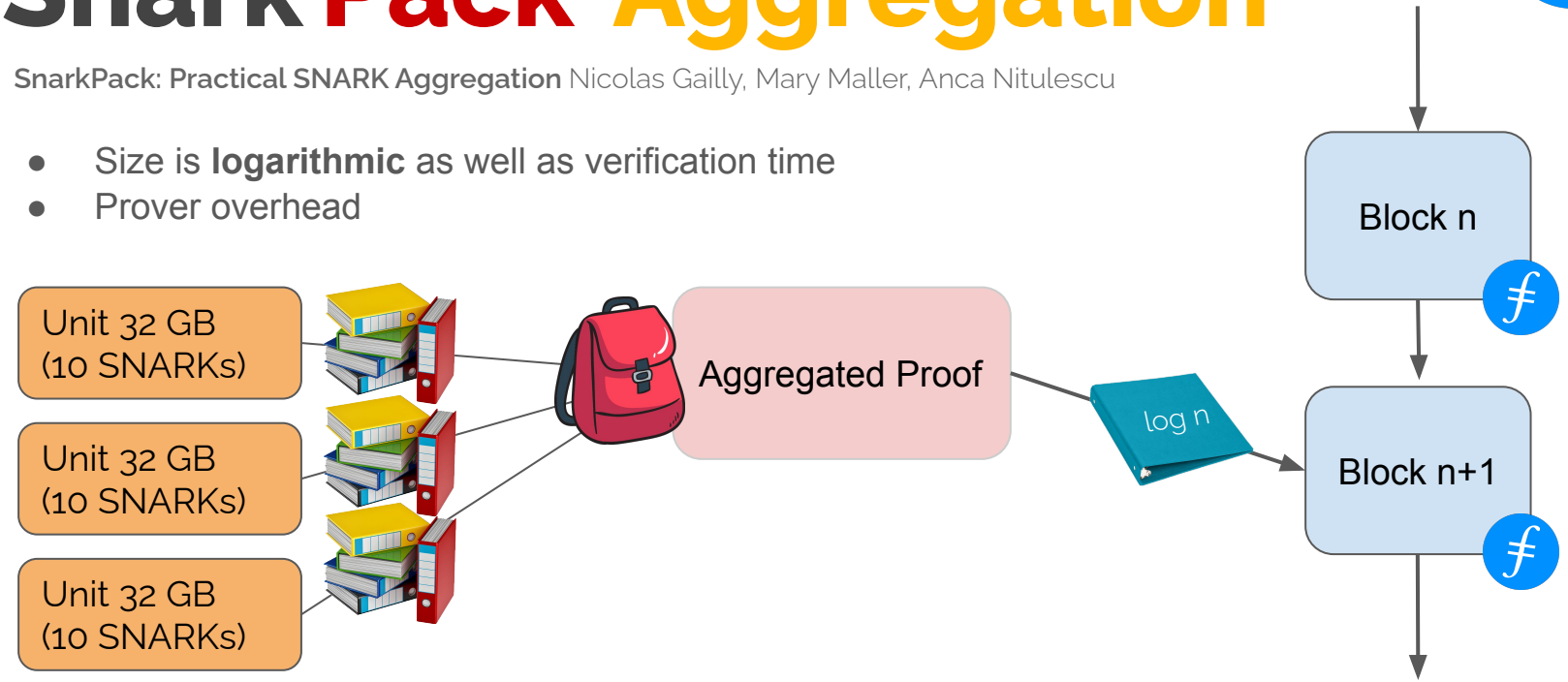
Proof of storage



Snark Pack Aggregation

SnarkPack: Practical SNARK Aggregation Nicolas Gailly, Mary Maller, Anca Nitulescu

- Size is **logarithmic** as well as verification time
- Prover overhead



Bilinear Groups

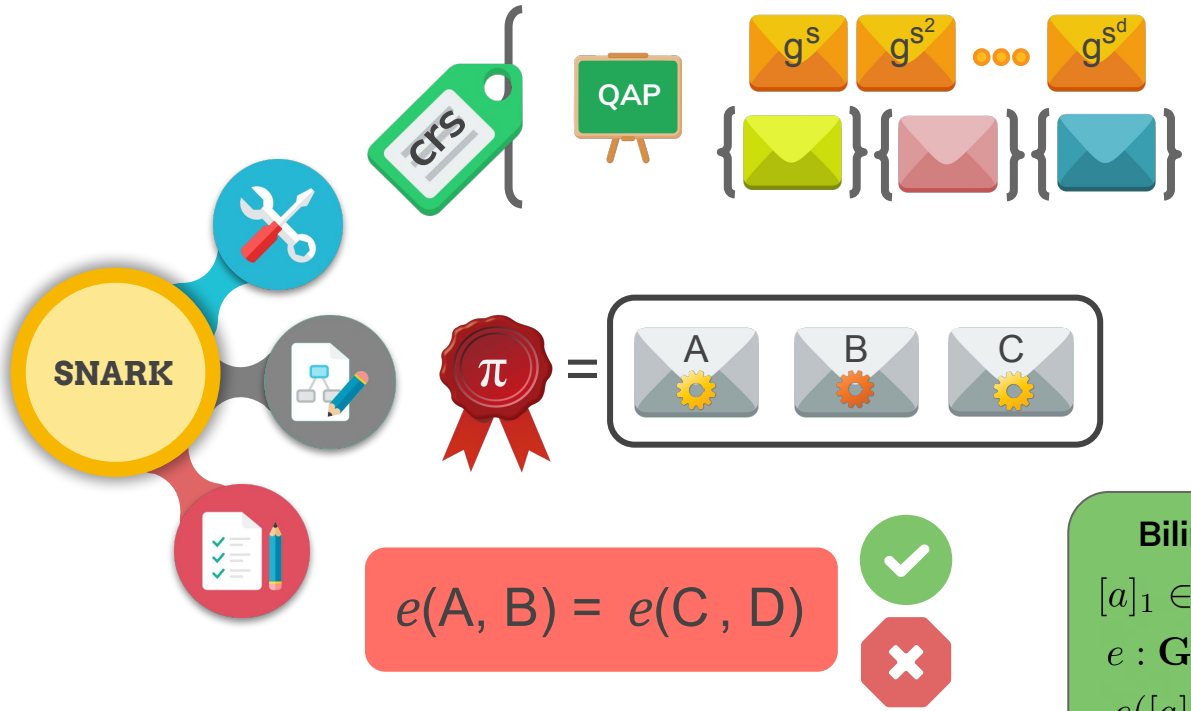
Bilinear Groups

$$[a]_1 \in \mathbf{G}_1, [b]_2 \in \mathbf{G}_2$$

$$e : \mathbf{G}_1 \times \mathbf{G}_2 \to \mathbf{G}_T$$

$$e([a]_1, [b]_2) = [ab]_T$$

Groth 16



Bilinear Groups

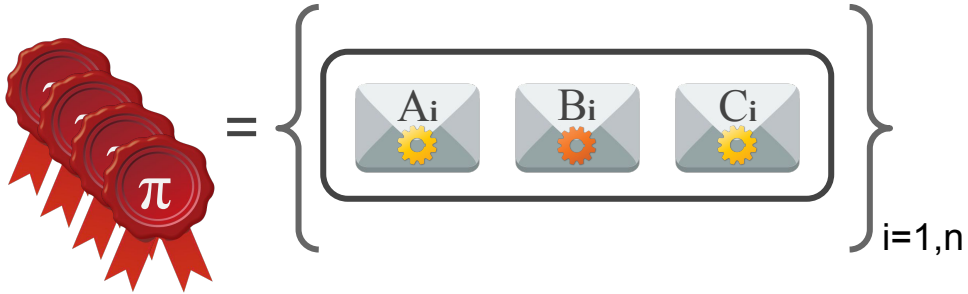
$[a]_1 \in \mathbf{G}_1, [b]_2 \in \mathbf{G}_2$

$e : \mathbf{G}_1 \times \mathbf{G}_2 \rightarrow \mathbf{G}_T$

$e([a]_1, [b]_2) = [ab]_T$

Many SNARKs

Proofs



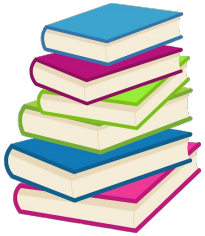
Verification ($D = g^d$)

$e(A_1, B_1) = e(C_1, D)$

$e(A_2, B_2) = e(C_2, D)$

...

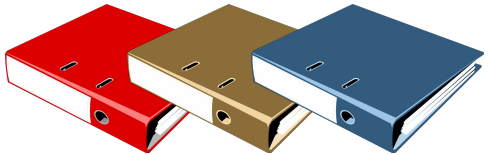
$e(A_n, B_n) = e(C_n, D)$



Verify many SNARKs

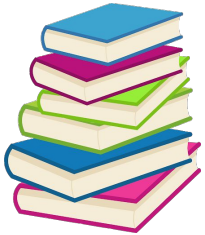
Batch Verification

Proof Size



Verification Time

SNARK Batching



Verification

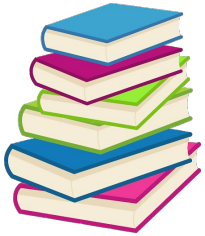
$e(A_1, B_1) = e(C_1, D)$

$e(A_2, B_2) = e(C_2, D)$

...

$e(A_n, B_n) = e(C_n, D)$

SNARK Batching



Verification

$e(A_1, B_1) = e(C_1, D)$

$e(A_2, B_2) = e(C_2, D)$

…

$e(A_n, B_n) = e(C_n, D)$

$r \leftarrow \$$

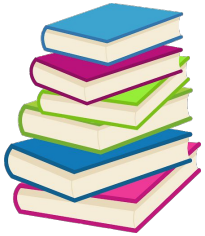
$r \times e(A_1, B_1) = e(C_1, D)$

$r^2 \times e(A_2, B_2) = e(C_2, D)$

…

$r^n \times e(A_n, B_n) = e(C_n, D)$

SNARK Batching



Verification

$e(A_1, B_1) = e(C_1, D)$

$e(A_2, B_2) = e(C_2, D)$

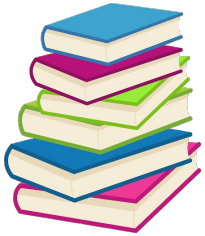
...

$e(A_n, B_n) = e(C_n, D)$

$r \leftarrow \$$

Batch Verification

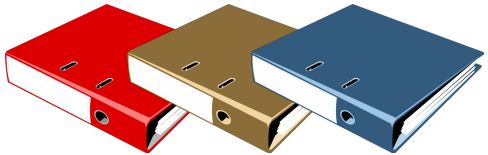
$$\sum r^i e(A_i, B_i) = \sum r^i e(C_i, D)$$



Verify many SNARKs

Batch Verification

Proof Size



Verification Time

Aggregation

Proof Size

Verification Time

SNARK Aggregation

SnarkPack: Practical SNARK Aggregation Nicolas Gailly, Mary Maller, Anca Nitulescu

Batch Verification

$$\sum e(A_i, r^i B_i) = \sum e(r^i C_i, D)$$

$$\Leftrightarrow$$

$$\sum e(A_i, r^i B_i) = e(\sum r^i C_i, D)$$

Bilinear Groups

$$[a]_1 \in \mathbb{G}_1, [b]_2 \in \mathbb{G}_2$$

$$e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$$

$$e([a]_1, [b]_1) = [ab]_T$$

SNARK Aggregation

SnarkPack: Practical SNARK Aggregation Nicolas Gailly, Mary Maller, Anca Nitulescu

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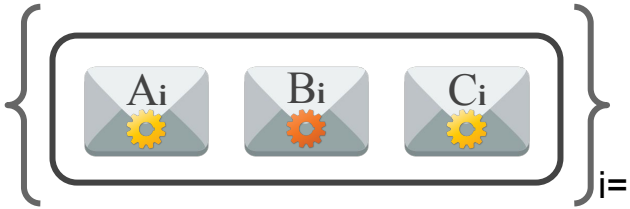
Aggregation



SNARK Aggregation

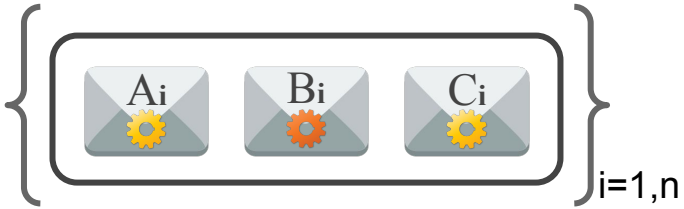
Batch Verification

$$\sum e(A_i, r^i B_i) = \sum e(r^i C_i, D)$$

$$\Leftrightarrow$$

$$\sum e(A_i, r^i B_i) = e(\sum r^i C_i, D)$$

Aggregation



$$Z_{AB} = \sum e(A_i, r^i B_i)$$

$$Z_C = \sum r^i C_i$$

SNARK Aggregation

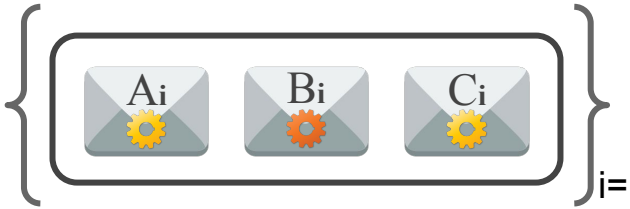
Batch Verification

$$\sum e(A_i, r^i B_i) = \sum e(r^i C_i, D)$$

$$\Leftrightarrow$$

$$Z_{AB} = e(Z_C, D)$$

Aggregation



$$Z_{AB} = \sum e(A_i, r^i B_i)$$

$$Z_C = \sum r^i C_i$$

SNARK Aggregation

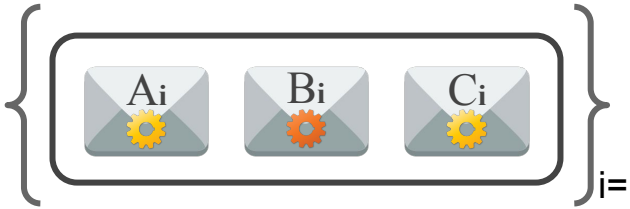
Batch Verification

$$\sum e(A_i, r^i B_i) = \sum e(r^i C_i, D)$$

$$\Leftrightarrow$$

$$Z_{AB} = e(Z_C, D)$$

Aggregation



$$Z_{AB} = \sum e(A_i, r^i B_i)$$

$$Z_C = \sum r^i C_i$$

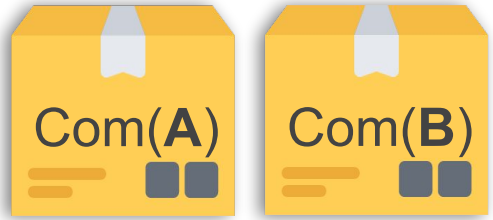
Tools: MIPP & TIPP

Proofs for Inner Pairing Products and Applications - Bünz, Maller, Mishra, Tyagi, Vesely

$\langle \mathbf{A}, \mathbf{r} \rangle = \sum r_i A_i$

$\langle \mathbf{A}, \mathbf{B} \rangle = \sum e(A_i, B_i)$

$A_i \in \mathbb{G}_1, B_i \in \mathbb{G}_2, r_i \in \mathbb{Z}_q$



Tools: MIPP & TIPP

Proofs for Inner Pairing Products and Applications - Bünz, Maller, Mishra, Tyagi, Vesely

$$\langle \mathbf{A}, \mathbf{r} \rangle = \sum r_i A_i = \sum [a_i, r_i]_1$$

$$\langle \mathbf{A}, \mathbf{B} \rangle = \sum e(A_i, B_i) = [\sum a_i b_i]_T$$

$$A_i \in \mathbb{G}_1, B_i \in \mathbb{G}_2, r_i \in \mathbb{Z}_q$$

Tools: MIPP & TIPP

Proofs for Inner Pairing Products and Applications - Bünz, Maller, Mishra, Tyagi, Vesely

$$\langle \mathbf{C}, \mathbf{r} \rangle = \sum r_i A_i$$

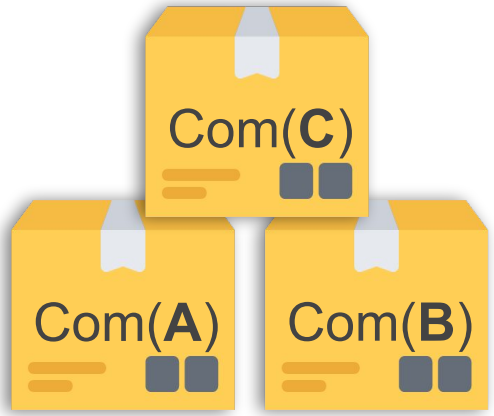
$$\langle \mathbf{A}, \mathbf{rB} \rangle = \sum e(A_i, B_i)$$

$$Z_C = \langle \mathbf{C}, \mathbf{r} \rangle$$

$$Z_{AB} = \langle \mathbf{A}, \mathbf{rB} \rangle$$

Aggregation

MIPP & TIPP Strategy



$Z_C = \langle C, r \rangle$

$Z_{AB} = \langle A, rB \rangle$

TIPP Folding Strategy

$\langle \mathbf{A}, \mathbf{B} \rangle = e(A_1, B_1) + e(A_2, B_2) \ldots + e(A_n, B_n)$

$\mathbf{A} = (A_1, A_2, \ldots A_n)$

$\mathbf{B} = (B_1, B_2 \ldots B_n)$

Split & Collapse

$$\langle \mathbf{A}, \mathbf{B} \rangle = e(A_1, B_1) + e(A_2, B_2) \ldots + e(A_n, B_n)$$

$\mathbf{A} = (A_1, A_2, \ldots A_n)$

$\mathbf{B} = (B_1, B_2 \ldots B_n)$

Split & Collapse

$$\langle \mathbf{A}, \mathbf{B} \rangle = e(A_1, B_1) + e(A_2, B_2) \ldots + e(A_n, B_n)$$

$\mathbf{A} = (A_1, A_2, \ldots A_n)$

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Split & Collapse

$$\langle \mathbf{A}, \mathbf{B} \rangle = e(A_1, B_1) + e(A_2, B_2) \ldots + e(A_n, B_n)$$

$$\mathbf{A} = (A_1, A_2, \ldots A_n)$$

$$\mathbf{B} = (B_1, B_2 \ldots B_n)$$

$\mathbf{A}_{left}$ $\mathbf{A}_{right}$ $\mathbf{B}_{left}$ $\mathbf{B}_{right}$

Split & Collapse

$$\langle \mathbf{A}, \mathbf{B} \rangle = e(A_1, B_1) + e(A_2, B_2) \ldots + e(A_n, B_n)$$

$$\mathbf{A} = (A_1, A_2, \ldots A_n) \qquad \mathbf{B} = (B_1, B_2 \ldots B_n)$$

$\mathbf{A}_{left}$ $\mathbf{A}_{right}$ $\mathbf{B}_{left}$ $\mathbf{B}_{right}$

$\mathbf{A'} = (A'_1, \ldots A'_{n/2})$ $\mathbf{B'} = (B'_1, \ldots B'_{n/2})$

Split & Collapse

$$\langle \mathbf{A}, \mathbf{B} \rangle = e(A_1, B_1) + e(A_2, B_2) \ldots + e(A_n, B_n)$$

$$\mathbf{A} = (A_1, A_2, \ldots A_n)$$

$$\mathbf{B} = (B_1, B_2 \ldots B_n)$$

$$\mathbf{A'} = (A'_1, \ldots A'_{n/2})$$

$$\mathbf{B'} = (B'_1, \ldots B'_{n/2})$$

Split & Collapse

$\langle \mathbf{A}, \mathbf{B} \rangle = e(A_1, B_1) + e(A_2, B_2) \ldots + e(A_n, B_n)$

$\mathbf{A} = (A_1, A_2, \ldots A_n)$

$\mathbf{B} = (B_1, B_2 \ldots B_n)$

$\langle \mathbf{A'}, \mathbf{B'} \rangle$

$\mathbf{A'} = (A'_1, \ldots A'_{n/2})$

$\mathbf{B'} = (B'_1, \ldots B'_{n/2})$

Split & Collapse

Split $A = (A_{left}, A_{right})$

$B = (B_{left}, B_{right})$

$Z'_{A,B} = \langle A_{half}, B_{half} \rangle$

$L = \langle A_{right}, B_{left} \rangle \qquad R = \langle A_{left}, B_{right} \rangle$

$Z'_{A,B} = L + \langle A, B \rangle + Rx$

$\langle A, B \rangle = e(A_1, B_1) + e(A_2, B_2) \ldots + e(A_n, B_n)$

Linear Verification

$Z_{AB}=\langle A, B\rangle$

$A=(A_1, A_2, \ldots A_n)$ → $A'=A_{left}A_{right}$ → ... → A_{final}

Linear Verification

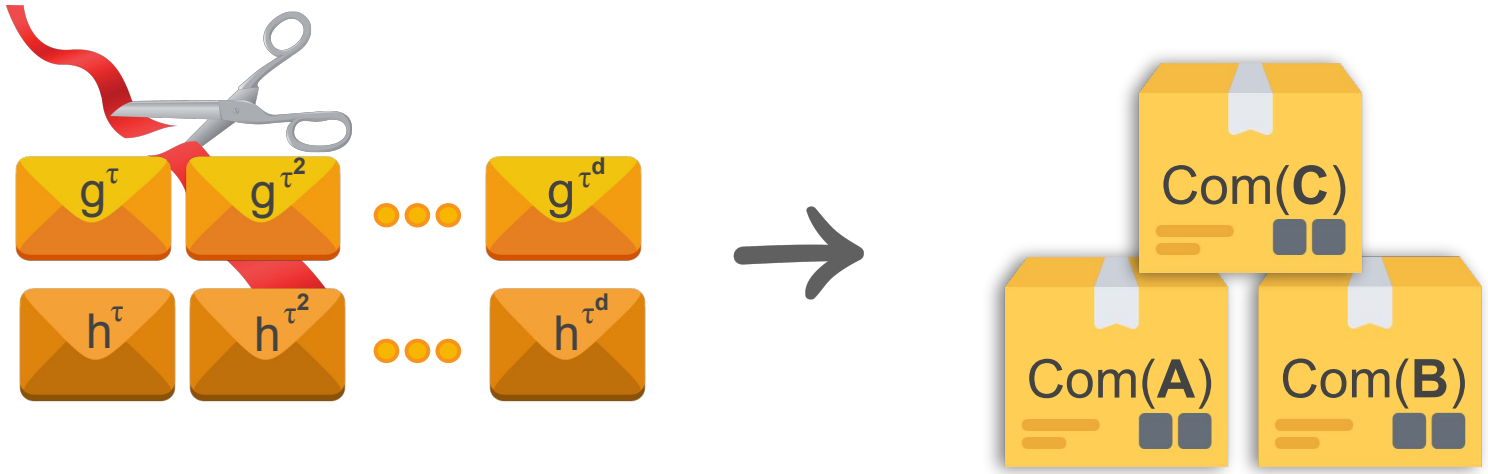
$Z_{AB}=\langle A, B\rangle$

$A=(A_1, A_2, \ldots A_n)$

$A'=A_{left}A_{right}$

A_{final}

Structured Setup



Vector Commitment

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Commit(ck, **a**) $\rightarrow$ 

$= [A(\tau)]_1 = A$

$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

Vector Commitment

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Commit(ck, **a**) $\rightarrow$  $= [A(\tau)]_1 = A$

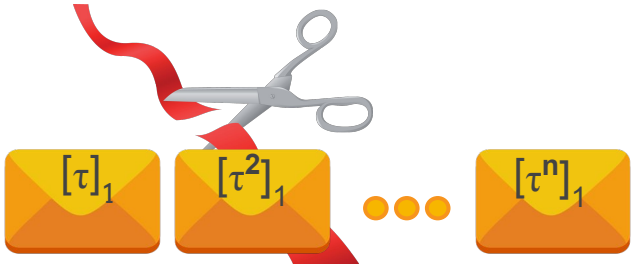
$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

Target-Group Commitment

A=  ... $\rightarrow$

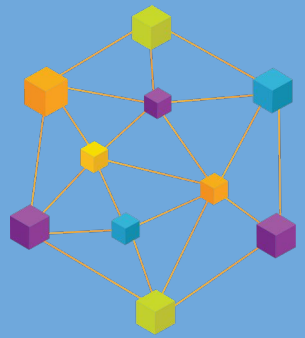
$Com(\mathbf{A}) = e(A_1,[\tau]_2)e(A_2,[\tau^2]_2)\ldots e(A_n,[\tau^n]_2)$

Structured Setup



Logarithmic verification for folding commitment key **ck**

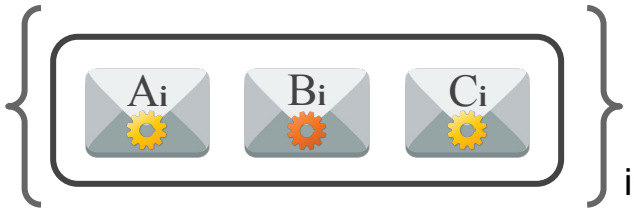
Fast verification ck_{final}: $g_\alpha(X) = \prod_{i=1}^{\mu} \left(1 + \alpha_{\mu+1-i}^{-2} X^{2^{i-1}}\right)$



Performance

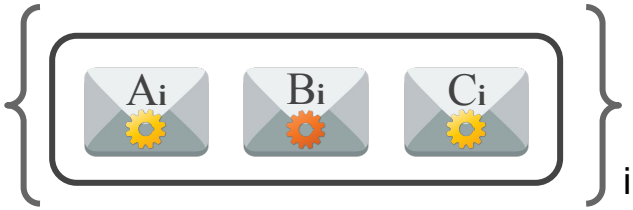
SNARK Aggregation

Aggregation



SNARK Aggregation

Aggregation



→

Com(**A, B**)

Com(**C**)

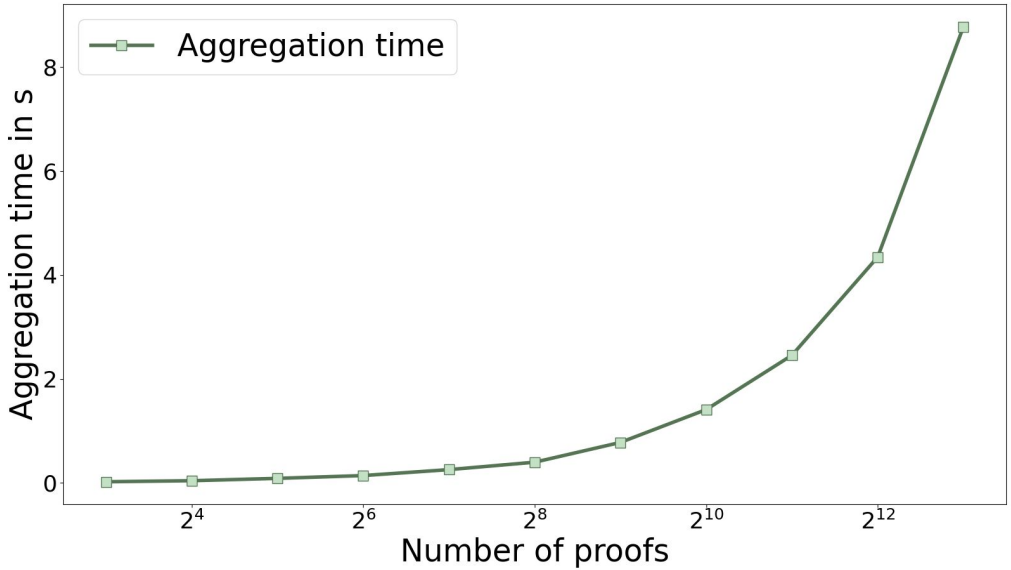
MIPP: $\langle \mathbf{C}, \mathbf{r} \rangle = \prod C_i^{r_i}$

TIPP: $\langle \mathbf{A}, \mathbf{B}^{\mathbf{r}} \rangle = \prod e(A_i, B_i^{r_i})$

→

log n

Aggregation

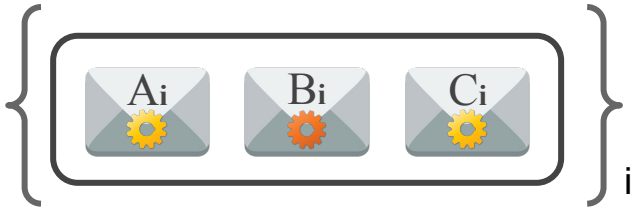


- 8.7s for 8 192 proofs
- Relies heavily on parallelism

- 2^{17} proofs in ~2mn

Proof size

Aggregation



→ Com(**A, B**) Com(**C**)

MIPP: $\langle \mathbf{C}, \mathbf{r} \rangle = \prod C_i^{r_i}$

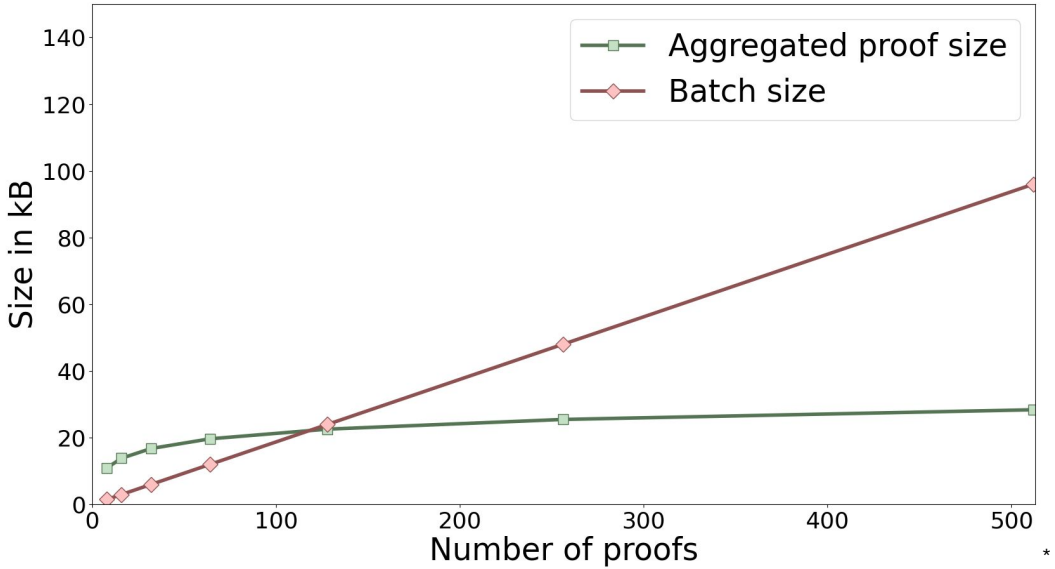
TIPP: $\langle \mathbf{A}, \mathbf{B}^{\mathbf{r}} \rangle = \prod e(A_i, B_i^{r_i})$

→

log n $Z_C = \langle \mathbf{C}, \mathbf{r} \rangle$

$Z_{AB} = \langle \mathbf{A}, \mathbf{B}^{\mathbf{r}} \rangle$

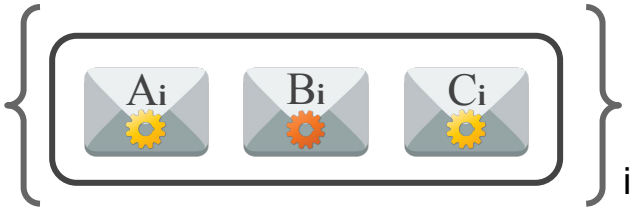
Proof size



- Use **compression** of target group points
  - based on Torus compression
  - credits - RELIC library implementation
- Turnover at 128 proofs
  - 23kB for aggregated
  - 24kB for “all proofs”

Verification

Aggregation



$Z_C = \langle C, r \rangle$

$Z_{AB} = \langle A, B^r \rangle$

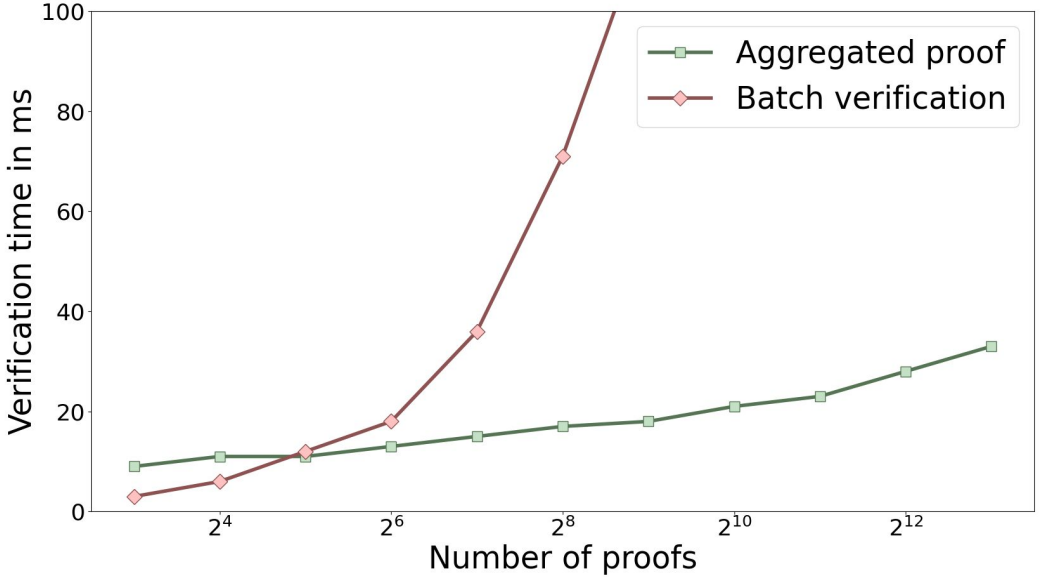
Verification

MIPP + TIPP

ck_{final}

$Z_{AB} = e(Z_C, D)$

Verifier Time



- Verifying aggregate proofs becomes **faster** from **32 proofs**
- **8192 proofs** in **33ms**
  - "ratio" of 0.004 ms per proof
- Optimizations:
  - Relies heavily on parallelism
  - MIPP/TIPP combined
  - Batching for pairing checks

SNARK Recursion

Prove many instances

Snark Pack

Recursion

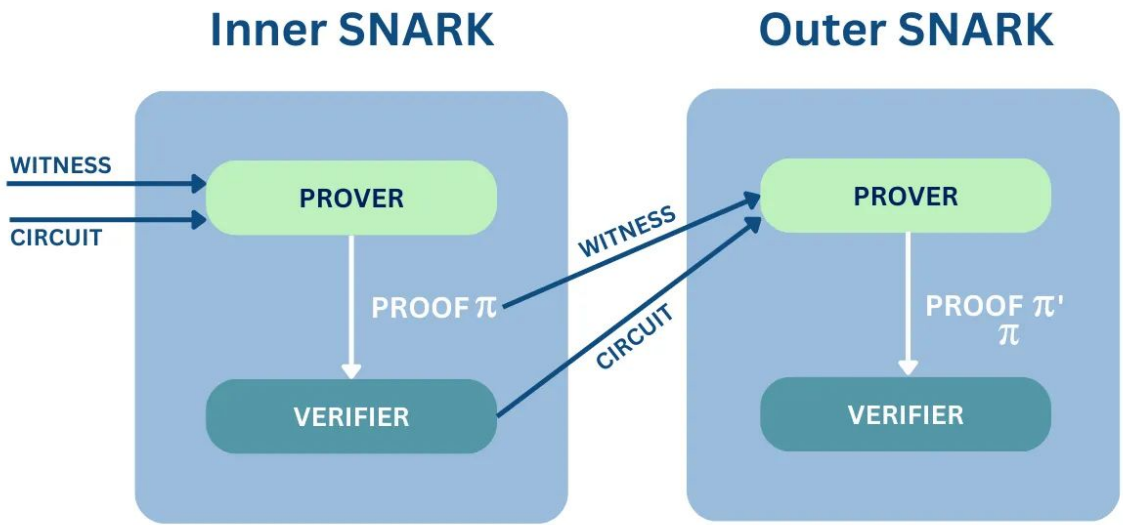
Folding

Proof Size

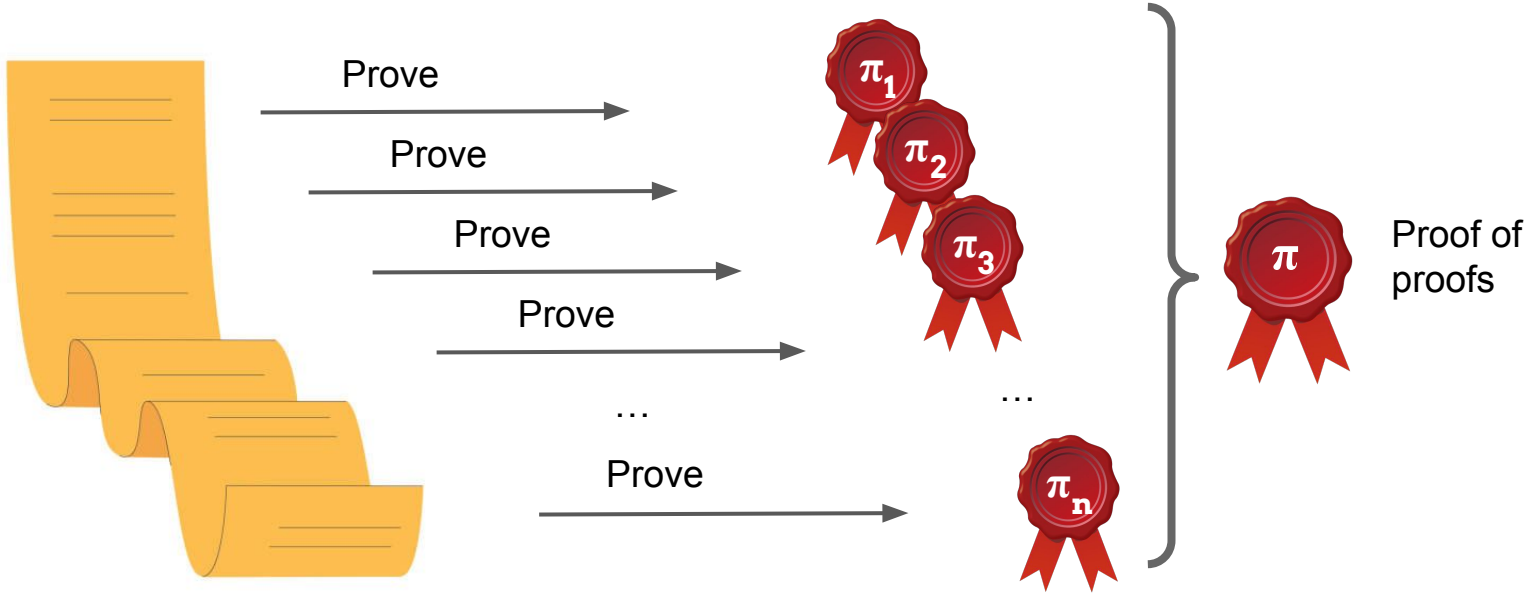
Verifier Time

Prover Time

Recursion

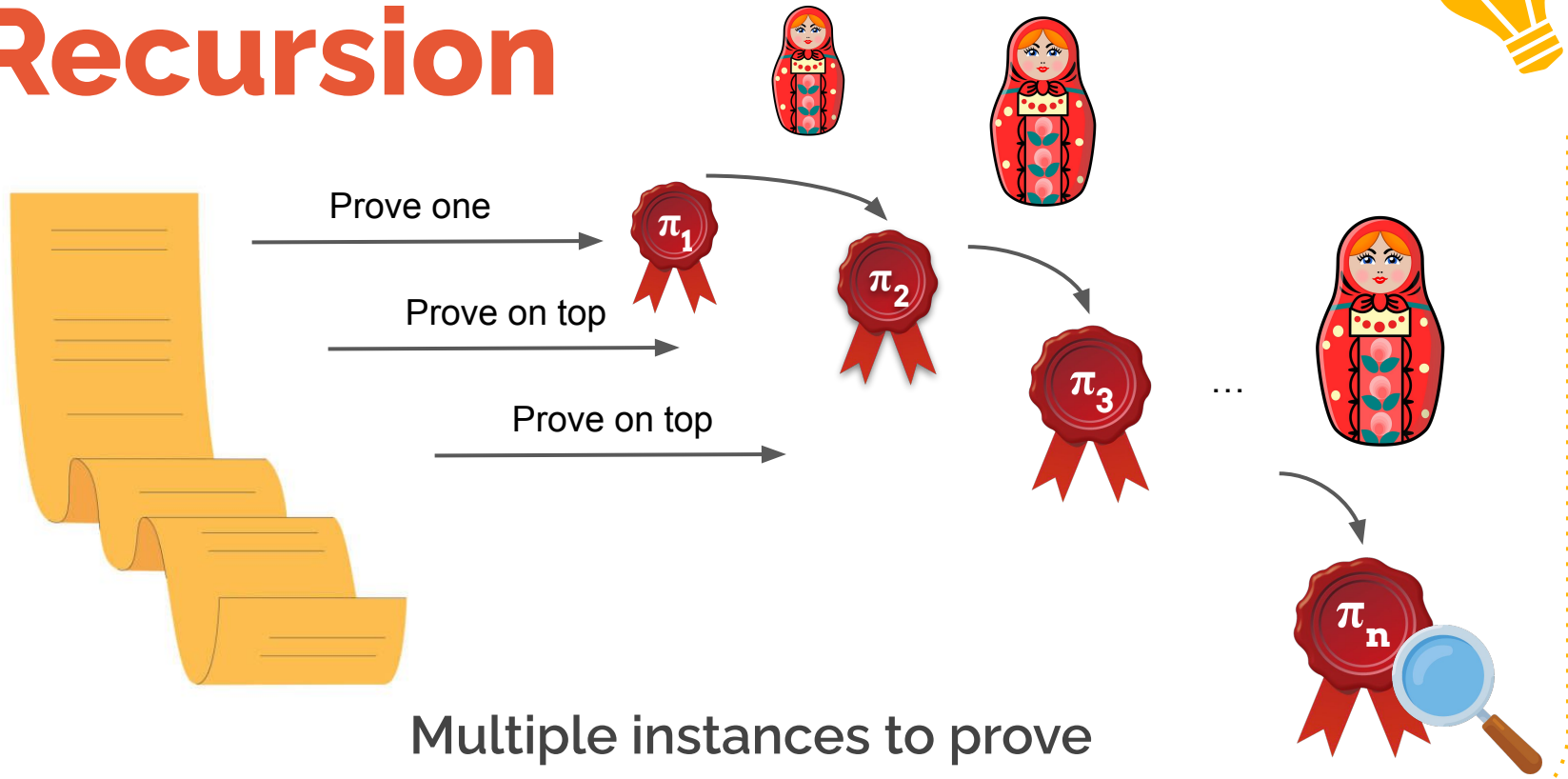

Inner SNARK
Outer SNARK
WITNESS
CIRCUIT
PROVER
PROOF π
VERIFIER
WITNESS
CIRCUIT
PROVER
PROOF π'
π
VERIFIER

Proof of proofs



Multiple instances to prove

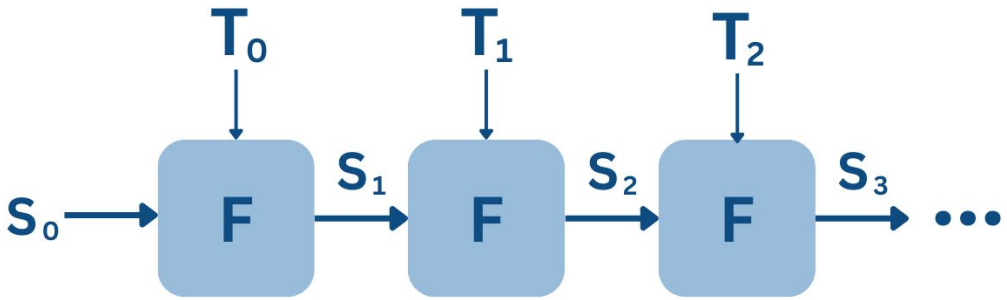
Recursion



Multiple instances to prove

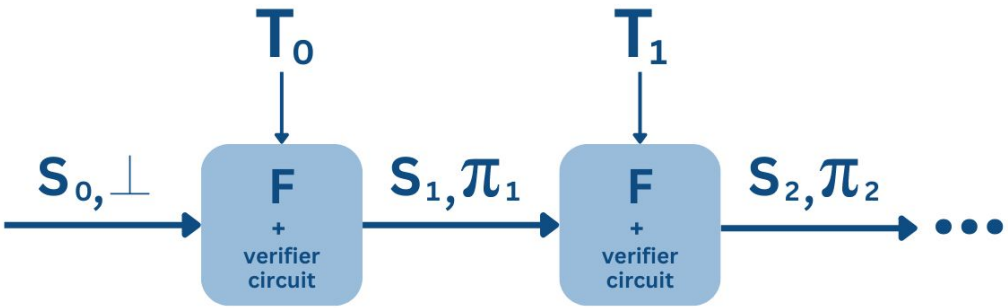
IVC

Incremental Verifiable Computation

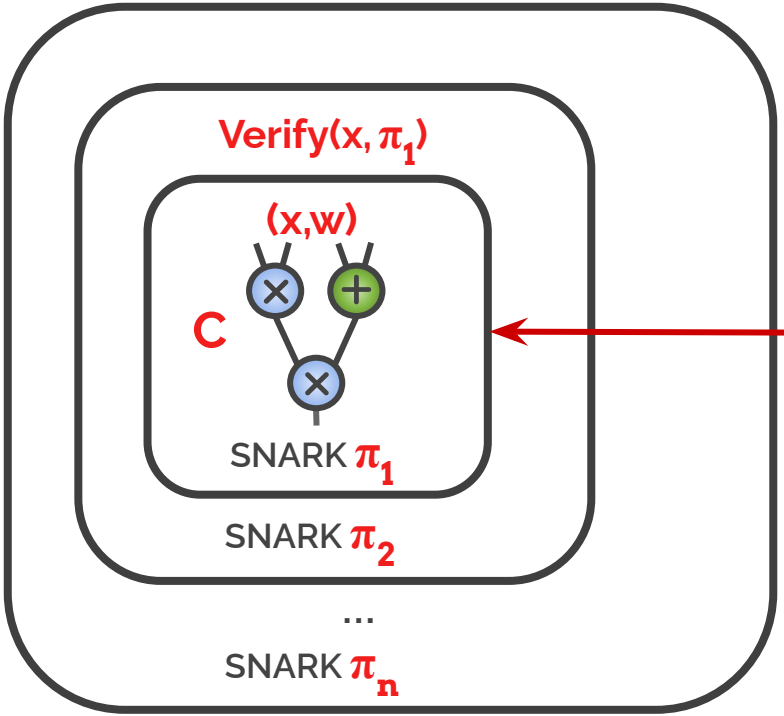


IVC

Incremental Verifiable Computation



Proof Recursion

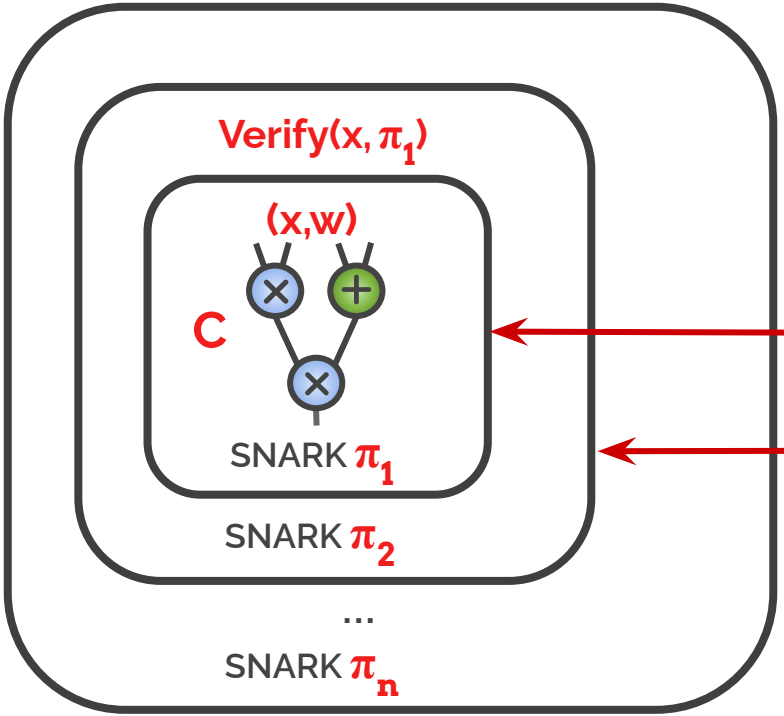


Verification Runtime

$|\text{Verify}| = 2|C_{\text{SNARK}}|$

$|C_{\text{Verify}_1}| = 2|C_{\text{SNARK}}|$

Proof Recursion



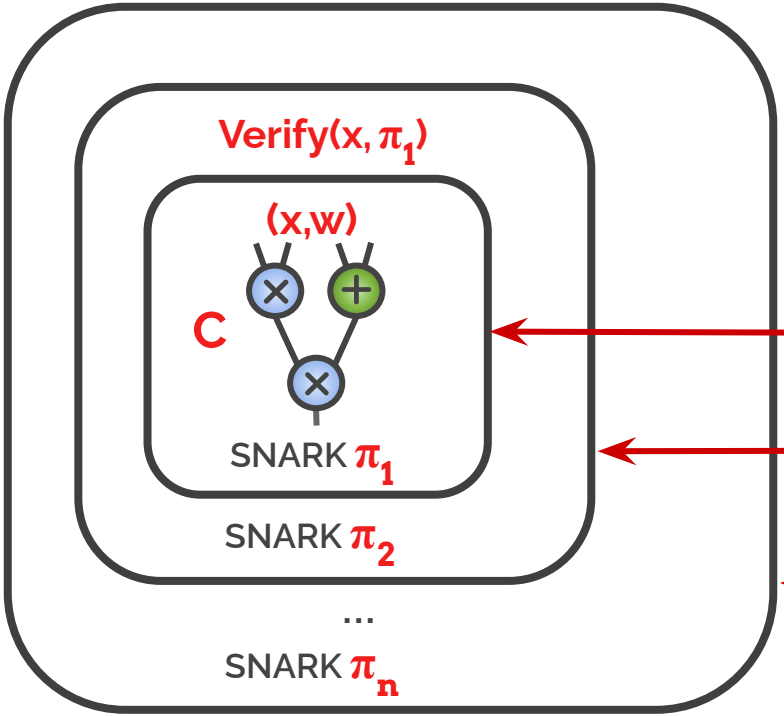
Verification Runtime

$|\text{Verify}| = 2|C_{\text{SNARK}}|$

$|C_{\text{Verify}_1}| = 2|C_{\text{SNARK}}|$

$|C_{\text{Verify}_2}| = 2|C_{\text{Verify}_1}| = 2^2|C_{\text{SNARK}}|$

Proof Recursion



Verification Runtime

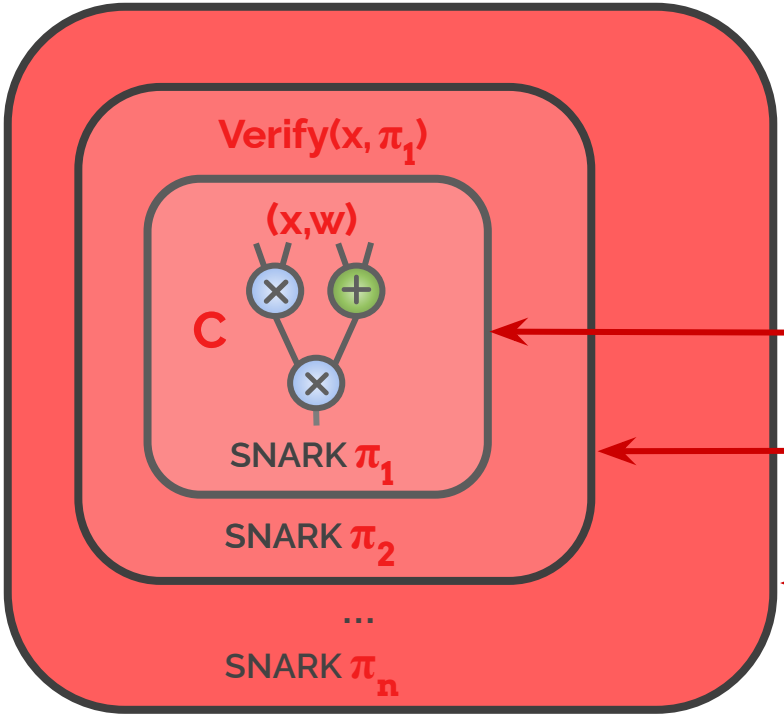
$|\text{Verify}| = 2|C_{\text{SNARK}}|$

$|C_{\text{Verify}_1}| = 2|C_{\text{SNARK}}|$

$|C_{\text{Verify}_2}| = 2|C_{\text{Verify}_1}| = 2^2|C_{\text{SNARK}}|$

$|C_{\text{Verify}_3}| = 2|C_{\text{Verify}_2}| = 2^3|C_{\text{SNARK}}|$

Proof Recursion



Needs Sublinear Verification!

$|C_{Verify_1}| = 2|C_{SNARK}|$

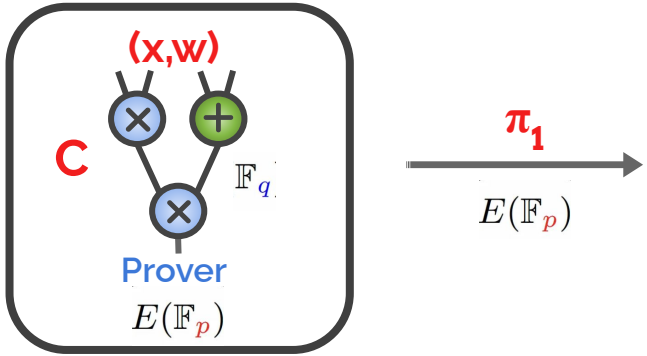
$|C_{Verify_2}| = 2|C_{Verify_1}| = 2^2|C_{SNARK}|$

$|C_{Verify_3}| = 2|C_{Verify_2}| = 2^3|C_{SNARK}|$

Proof Recursion

Verification Arithmetization

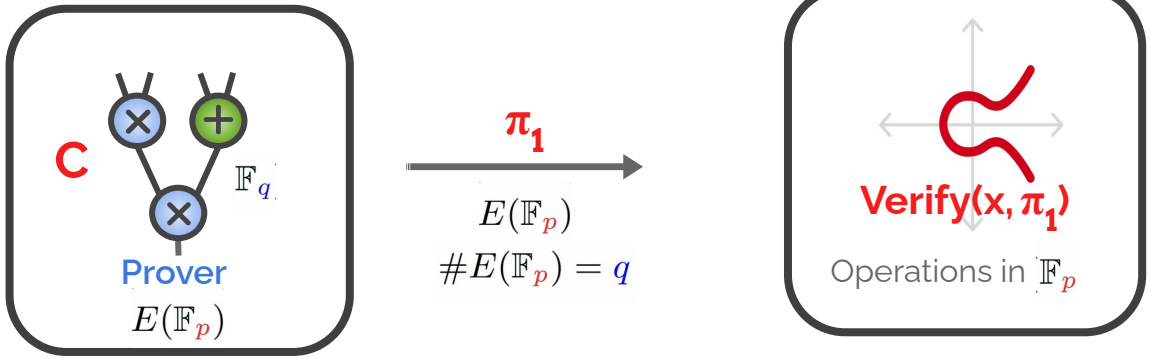
Verify = curve operations



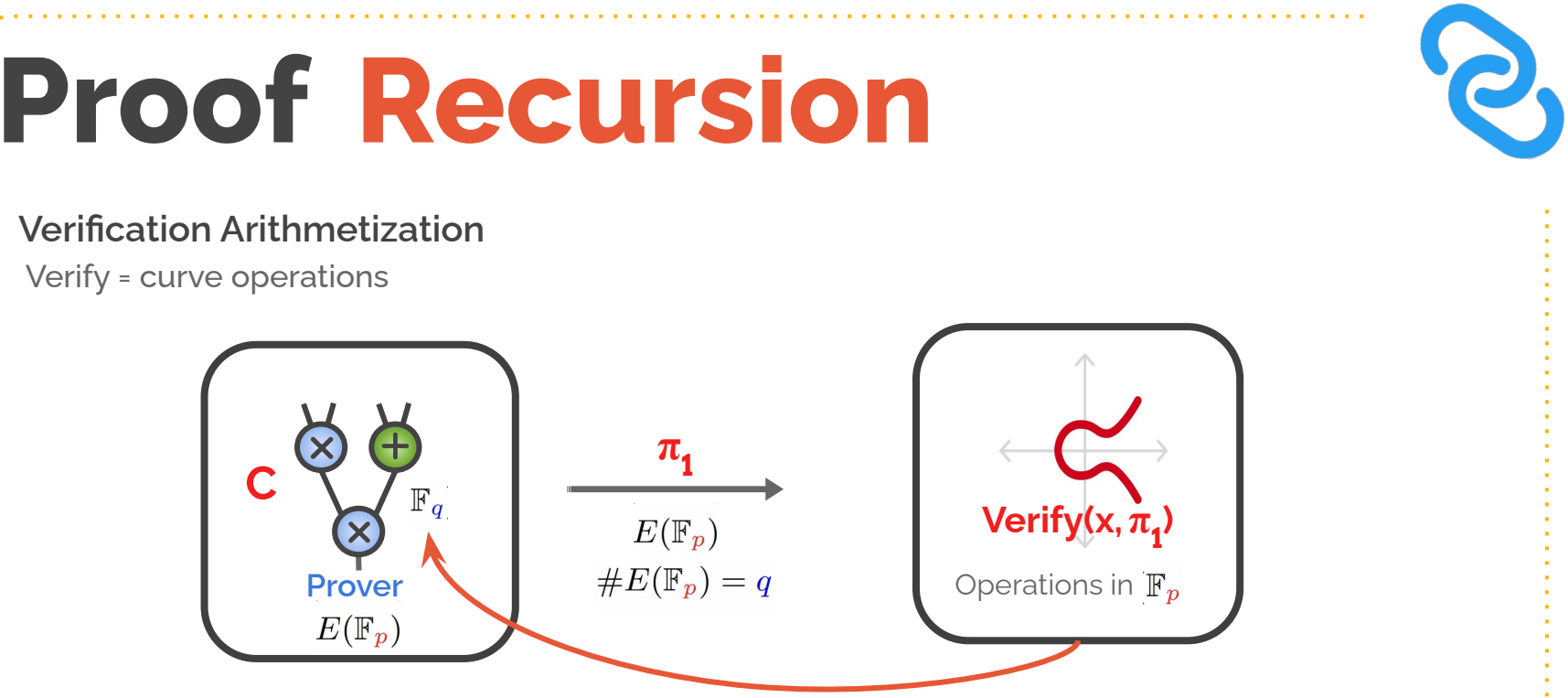
Proof Recursion

Verification Arithmetization

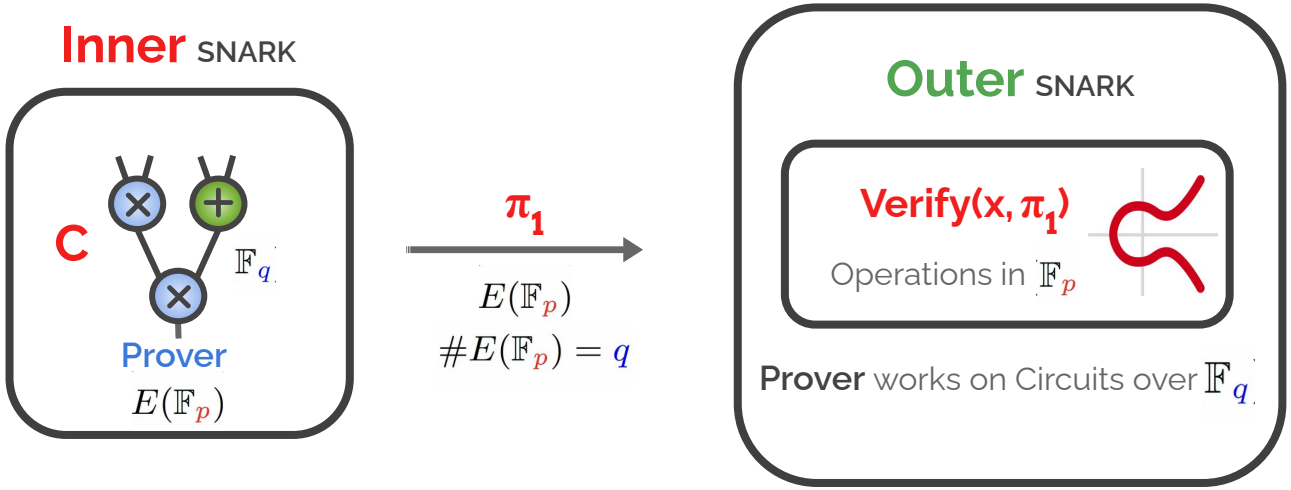
Verify = curve operations



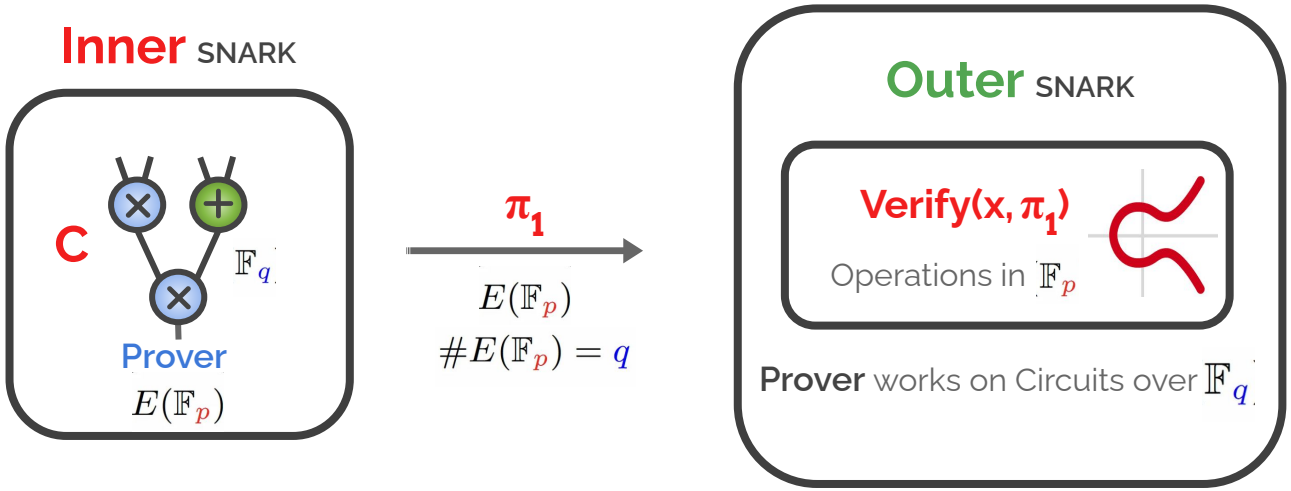
Proof Recursion



Proof Recursion



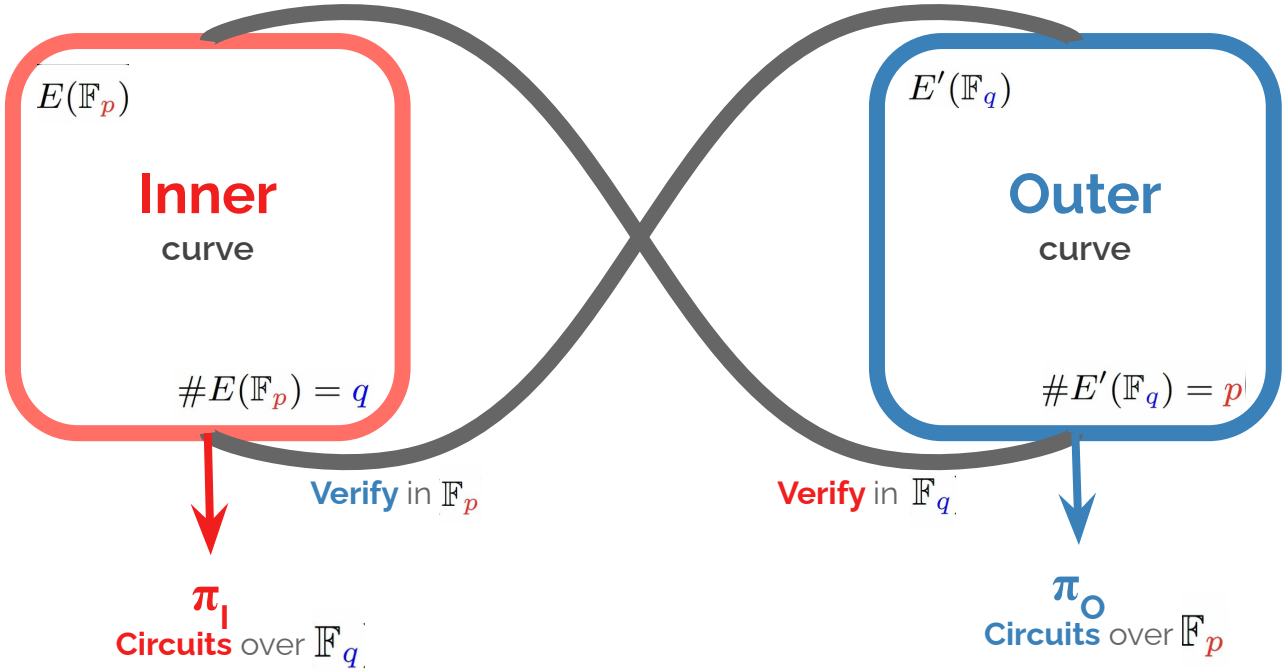
Proof Recursion



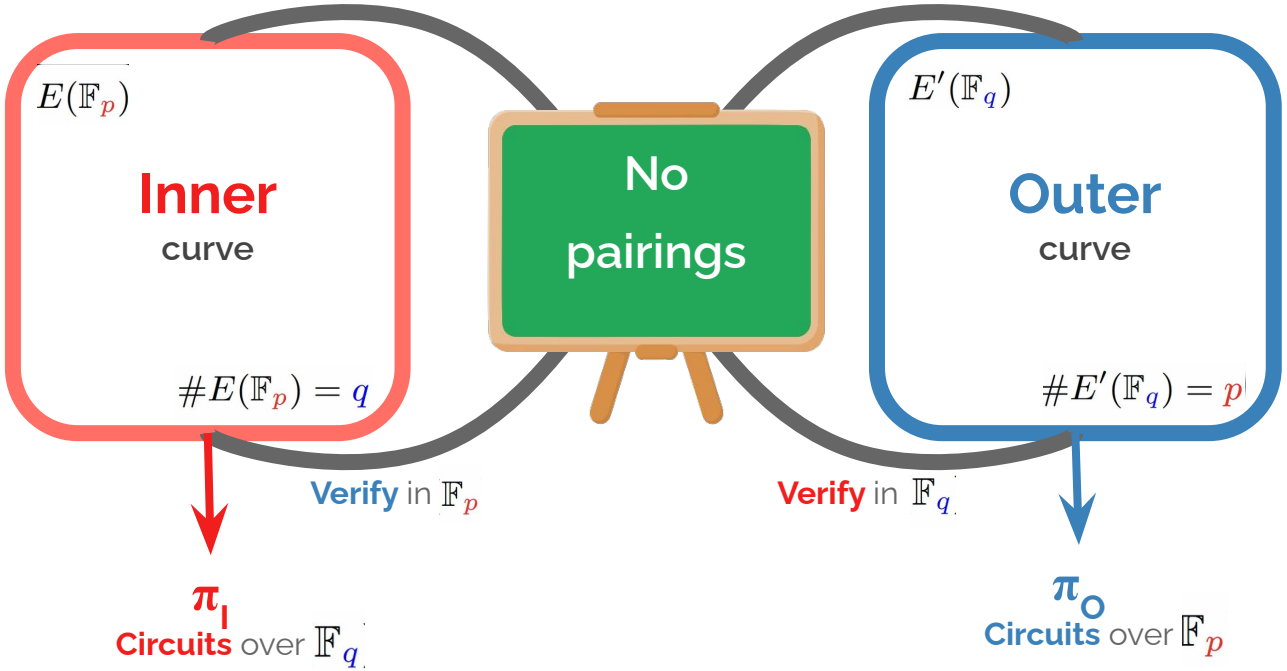
p = q

Dlog is easy to solve :(

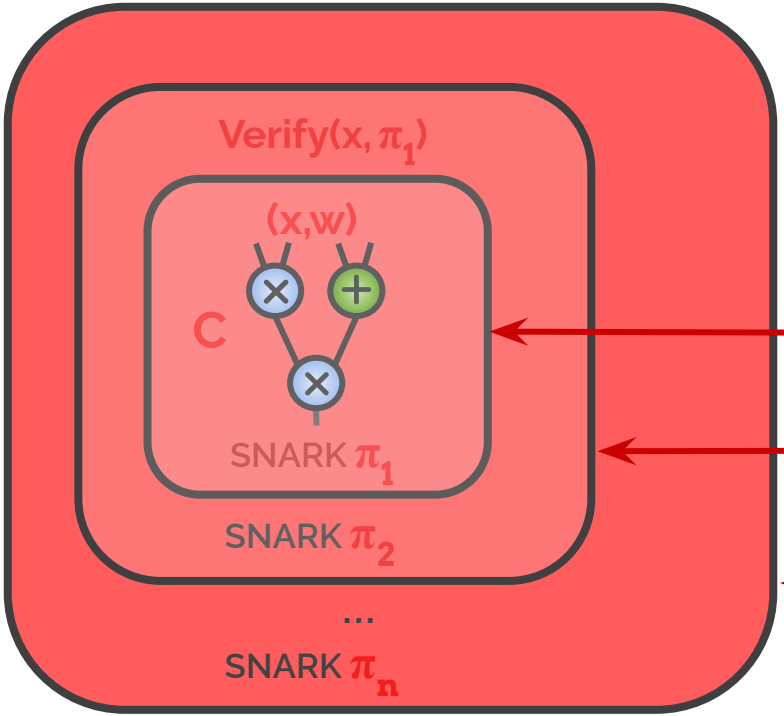
Cycles of Curves



Cycles of Curves



Linear Verfication!!!

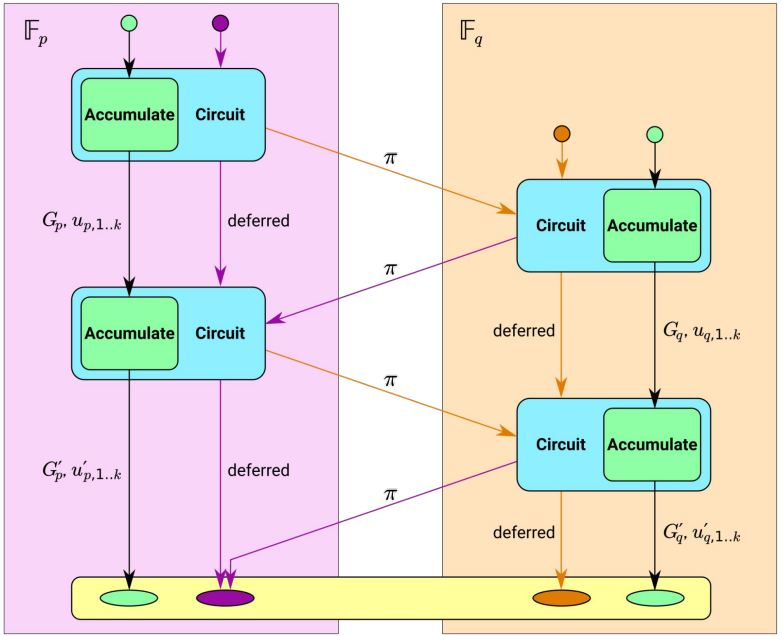


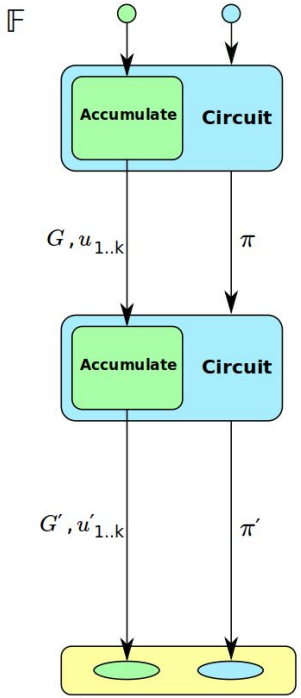
$|C_{Verify_1}| = 2|C_{SNARK}|$

$|C_{Verify_2}| = 2|C_{Verify_1}| = 2^2|C_{SNARK}|$

$|C_{Verify_3}| = 2|C_{Verify_2}| = 2^3|C_{SNARK}|$

Halo Roadmap





Source: [The Halo2 Book]

Thanks

https://www.di.ens.fr/~nitulesc/

Credits

Special thanks to all those who made and released these resources for free:

- Presentation template by SlidesCarnival
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- Illustrations by Disneyclips
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