Approximation Errors of Discretizations of the Radon Transform

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The Radon transform is a cornerstone of tomographic imaging applications. The concrete solution of tomographic inversion requires discretization of the Radon transform and its adjoint – the backprojection. Motivated by heuristics to address specific properties, many discretization approaches were proposed. It is common to use different discretization approaches for the forward operator and the backprojection, creating an unmatched (non-adjoint) pair of operators. Since iterative methods assume the usage of an operator and its adjoint, deviation from adjointess might harm the convergence. The well-known “Pixel-Driven’ approach is frequently used for the backprojection but anecdotally known to be ill-suited for approximation to the forward operator, showing distinct oscillation artifacts. The word “anecdotal’ reveals that there is very little rigorous mathematical analysis of such methods. We present such analysis for Pixel-Driven methods, showing rates for the approximation error in the operator norm between L2 spaces. These rates inform about suitable strategies for the discretization of the occurring variables, in particular, showing that the standard strategy with the same resolution for detector and image is not adequate, thus explaining the said artifacts.