Three balls inequalities for Schrödinger operators on periodic graphs

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Abstract

Three ball inequalities are a useful tool in the study of unique continuation properties in the continuum. However, in the discrete setting, these unique continuation properties do not hold. It has been shown that for the Laplace operator in $h\mathbb{Z}^d$ it is possible to correct the three ball inequality by adding an error term that decreases exponentially as h, the size of the lattice, tends to zero. Our goal is to extend this inequality to other types of lattices, known as periodic graphs. Periodic graphs are graphs in $\mathbb{R}^d$ that remain invariant under translations by vectors from a lattice $\mathcal{L} \subset \mathbb{R}^d$. Some examples are the Square lattice, the Triangular lattice, and the Hexagonal lattice.

Our main tool towards the proof of three ball inequalities is a Carleman-type estimate on a family of periodic graphs which allows us to deduce three balls inequalities for Schrödinger operator on this family. It also allows us to extend the inequalities the Laplace operator, using harmonicity, to a wider family of periodic graphs.