Continuum limit of discrete Anderson Hamiltonian in dimensions 2 and 3

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The discrete Anderson Hamiltonian is a Schrödinger operator introduced by physicists in order to model a conducting environment (e.g., metallic sample) perturbed by impurities. Mathematically, it is a random bounded operator on the lattice given by the sum of a discrete Laplacian and i.i.d. noises. On a formal level, as the mesh size of the lattice goes to zero, the operator converges to the continuous Anderson Hamiltonian, which is given by the sum of the continuous Laplacian and the Gaussian white noise. The latter operator is an unbounded one whose construction is non-trivial and requires machineries from singular SPDE theories. There have been considerable amount of recent results focusing on construction of the continuous operator and its spectral properties, but the convergence of the discrete to the continuous Anderson Hamiltonian remains formal: as the two operators live on different Hilbert spaces, it is a priori unclear how one can compare them and introduce a notion of convergence. The present talk has for objective to bridge this gap. We claim that the Anderson Hamiltonian defined on a discrete torus converges in Weidmann's generalised norm-resolvent sense to the continuous Anderson Hamiltonian defined on a continuous torus. As a corollary, the spectrum of the former operator converges to that of the latter. A key ingredient of the proof is the equivalent conditions for generalised norm-resolvent convergence formulated by Post and Zimmer, as well as the solution theory for the parabolic equations associated with these operators.