Title: Discrete Painlevé equations and pencils of quadrics in P^3

Yuri Suris

Abstarct: Discrete Painlevé equations constitute a famous class of integrable non-autonomous second order difference equations. A classification scheme proposed by Sakai interprets a discrete Painlevé equation as a birational map between generalized Halphen surfaces (surfaces obtained from P^1 × P^1 by blowing up at eight points). We propose a novel geometric interpretation of discrete Painlevé equations, where the family of generalized Halphen surfaces is replaced by a pencil of quadrics in P^3. In our scheme, discrete Painlevé equations are viewed as deformations of 3D QRT maps, defined geometrically as a composition of involutions along generators of quadrics of a pencil, preserving the intersection curves with a second pencil of quadrics. The base set of the net of quadrics spanned by both pencils consists of eight points (which play the role of the eight blow-up points of generalized Halphen surfaces). A Painlevé deformation of a 3D QRT map is obtained by composing the above mentioned involutions with a transformation of P^3 under which the pencil remains invariant, but the individual quadrics are mapped according to a certain transformation of the pencil parameter (dictated by the geometry of the base set of the pencil). Based on a joint work with J. Alonso and Kangning Wei.