UNIVERSITY OF OXFORD
Mathematical Institute

Heterogeneity in soft matter
Does it matter?

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Motivation

Tendons

Tissues connecting muscle to bone

Withstand large tensile loads

Tendinopathy

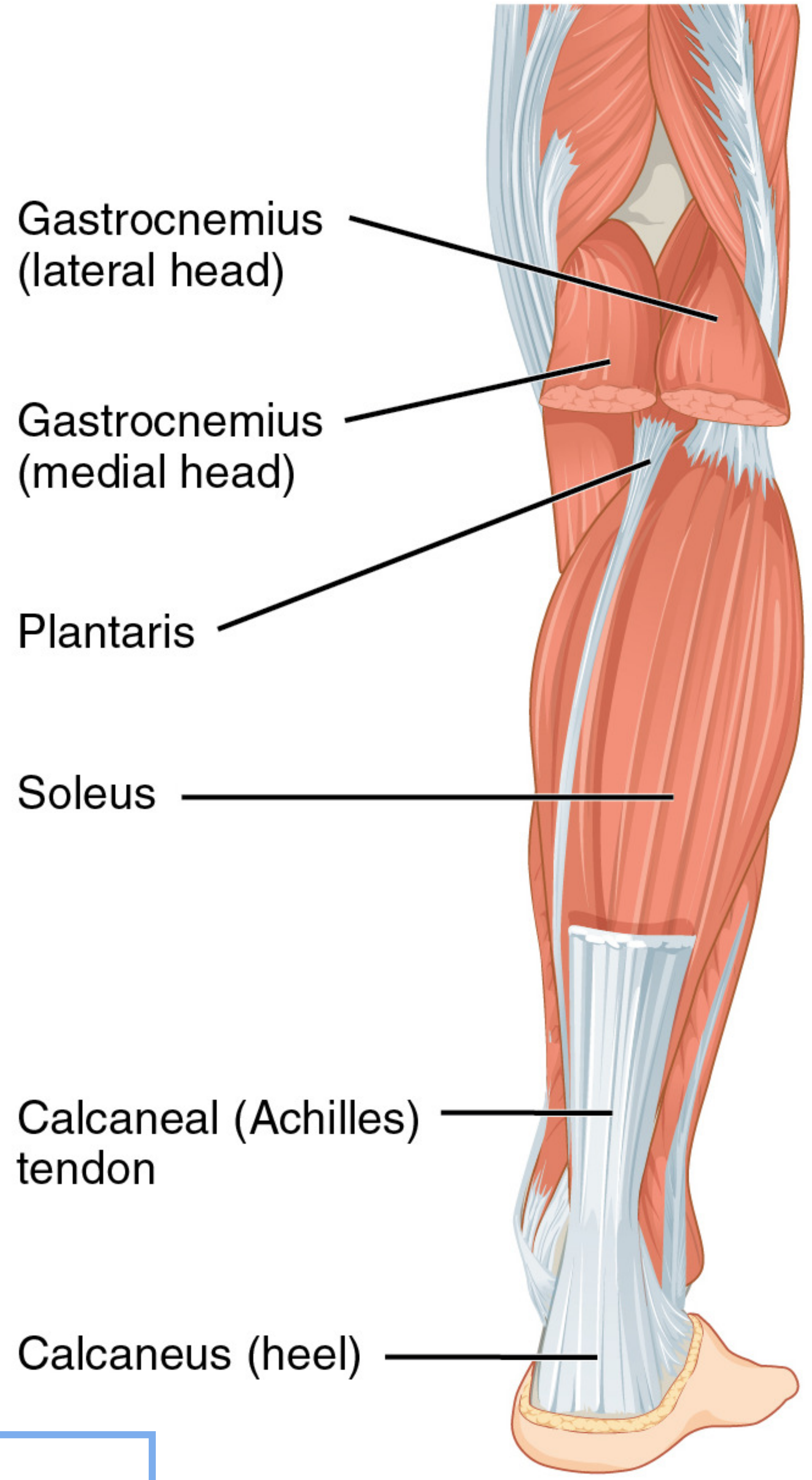
Common, painful condition

Overuse injury / disease

Altered tissue **structure**, **composition** and **volume**

Model:

Non-linear poroelasticity with material heterogeneity



Wong M, Jardaly AH, Kiel J. Anatomy, Bony Pelvis and Lower Limb: Achilles Tendon. [Updated 2023 Aug 8]. In: StatPearls [Internet]

How does material heterogeneity affect the response of a porous material to cyclical pulling?

Non-linear poroelasticity

(in Lagrangian)

$$\frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial Z}\left[\frac{k(\Phi)}{1+\Phi-\Phi_0}\frac{\partial p}{\partial Z}\right], \qquad \frac{\partial p}{\partial Z} = \frac{\partial s'}{\partial Z}$$

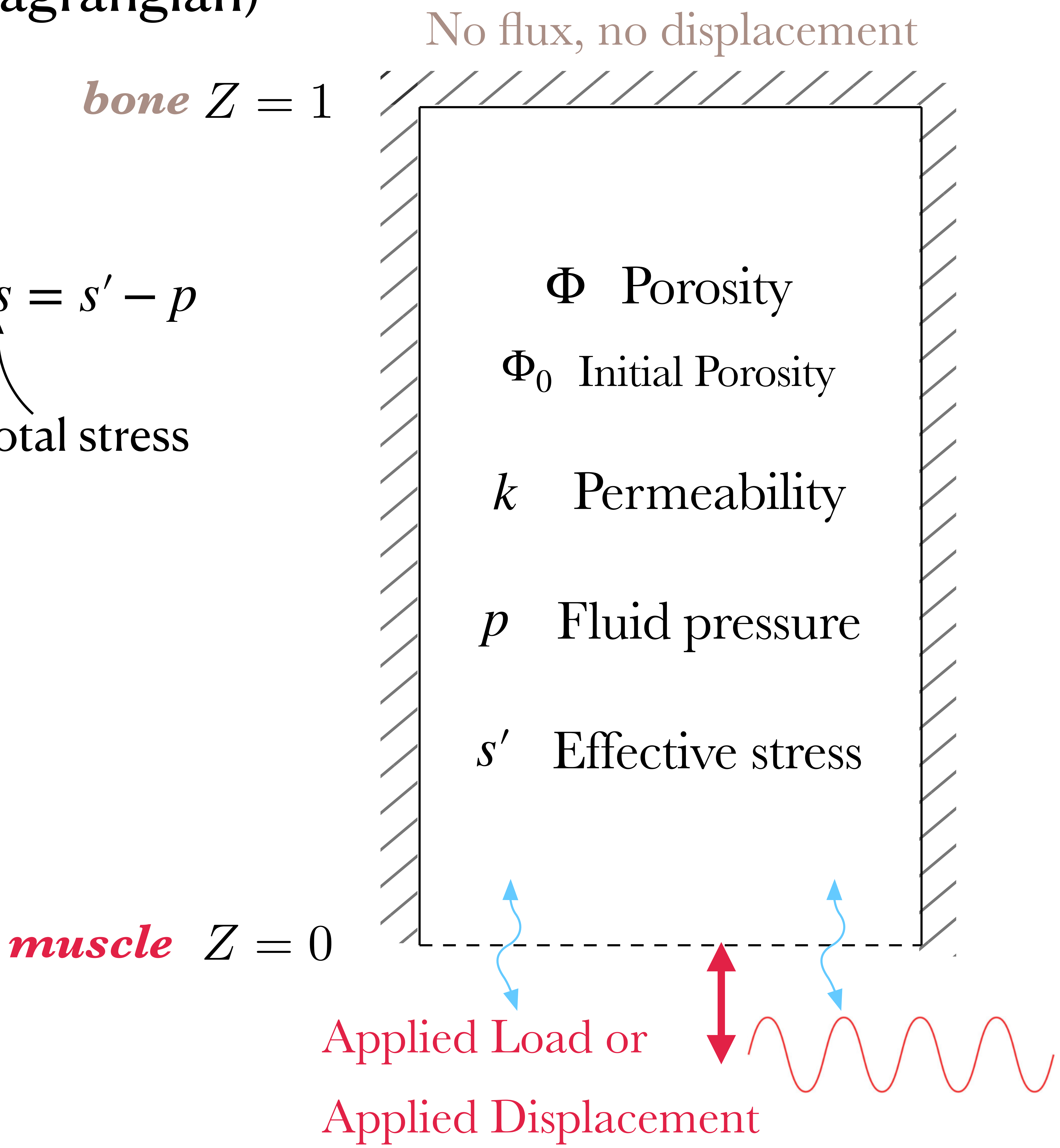
Conservation of mass

Darcy flow

Mechanical equilibrium

$$s = s' - p$$

total stress

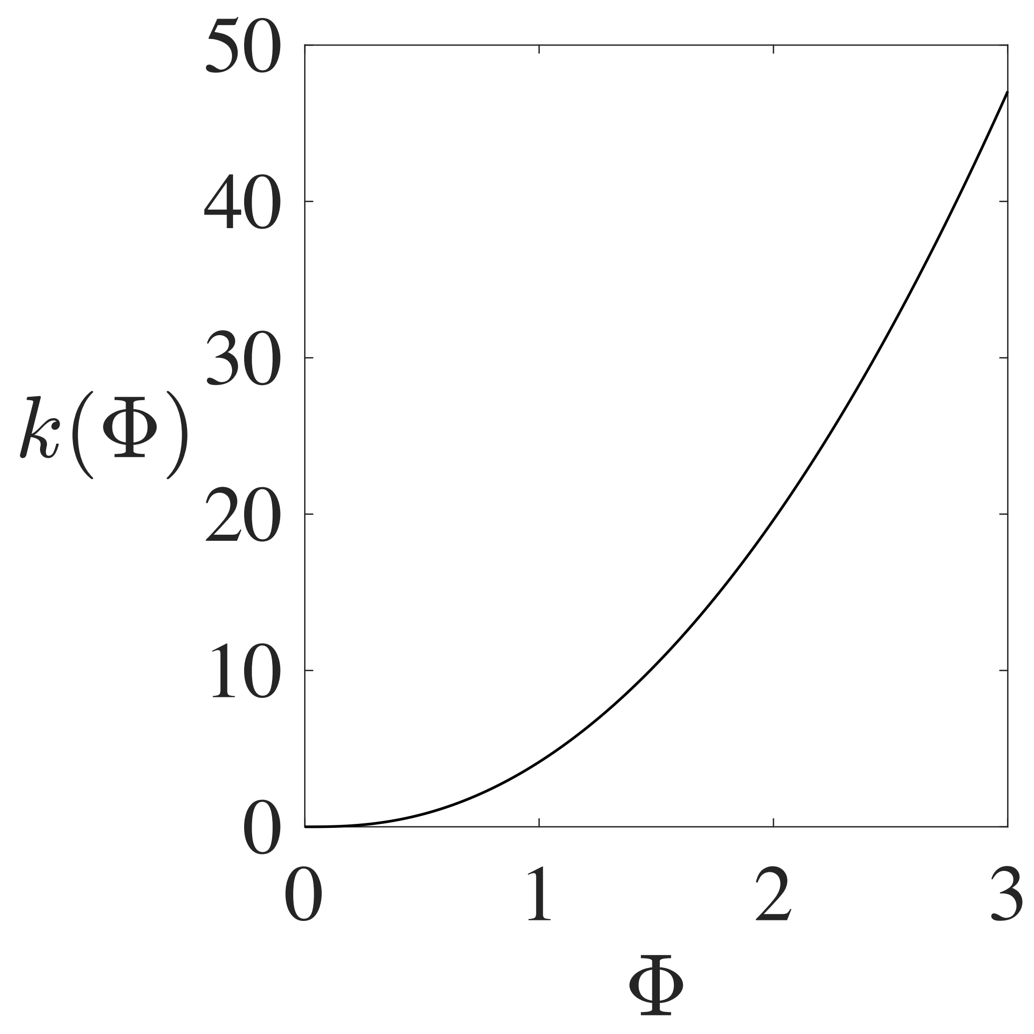


Non-linear poroelasticity

(in Lagrangian)

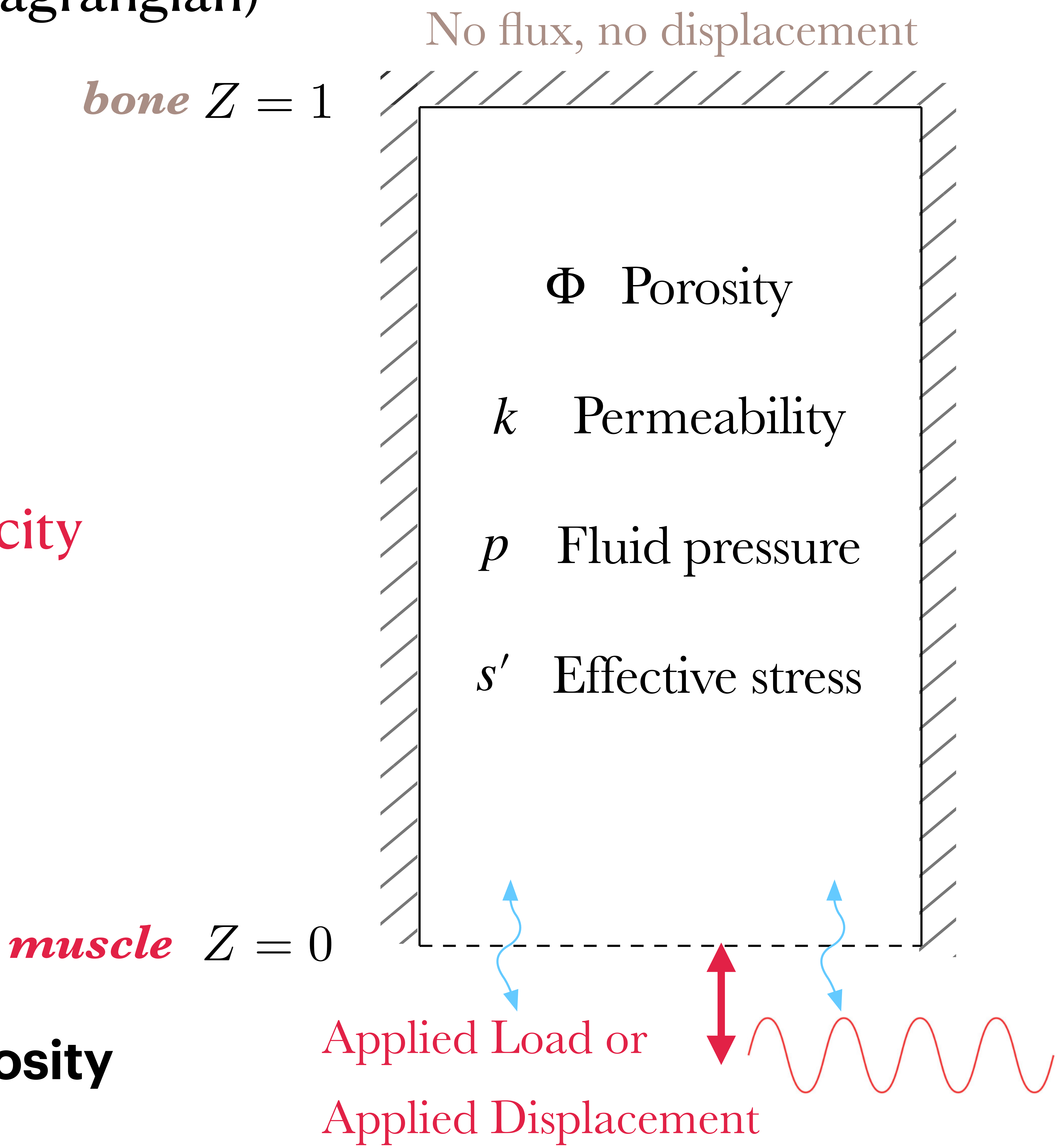
Kozeny-Carman permeability

$$\frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial Z}\left[\frac{k(\Phi)}{1+\Phi-\Phi_0}\frac{\partial p}{\partial Z}\right], \quad \frac{\partial p}{\partial Z} = \frac{\partial s'}{\partial Z}$$



Neo-Hookean elasticity

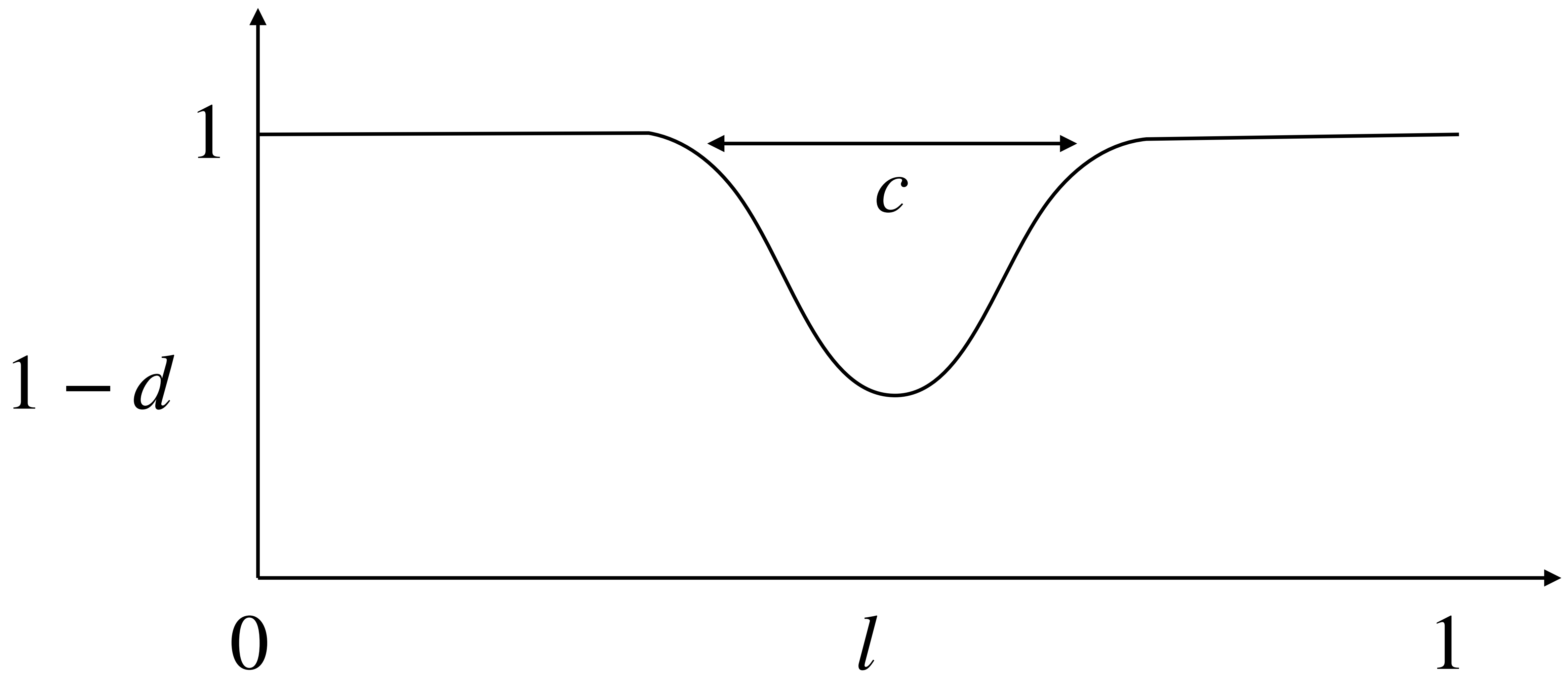
$s'(\Phi)$



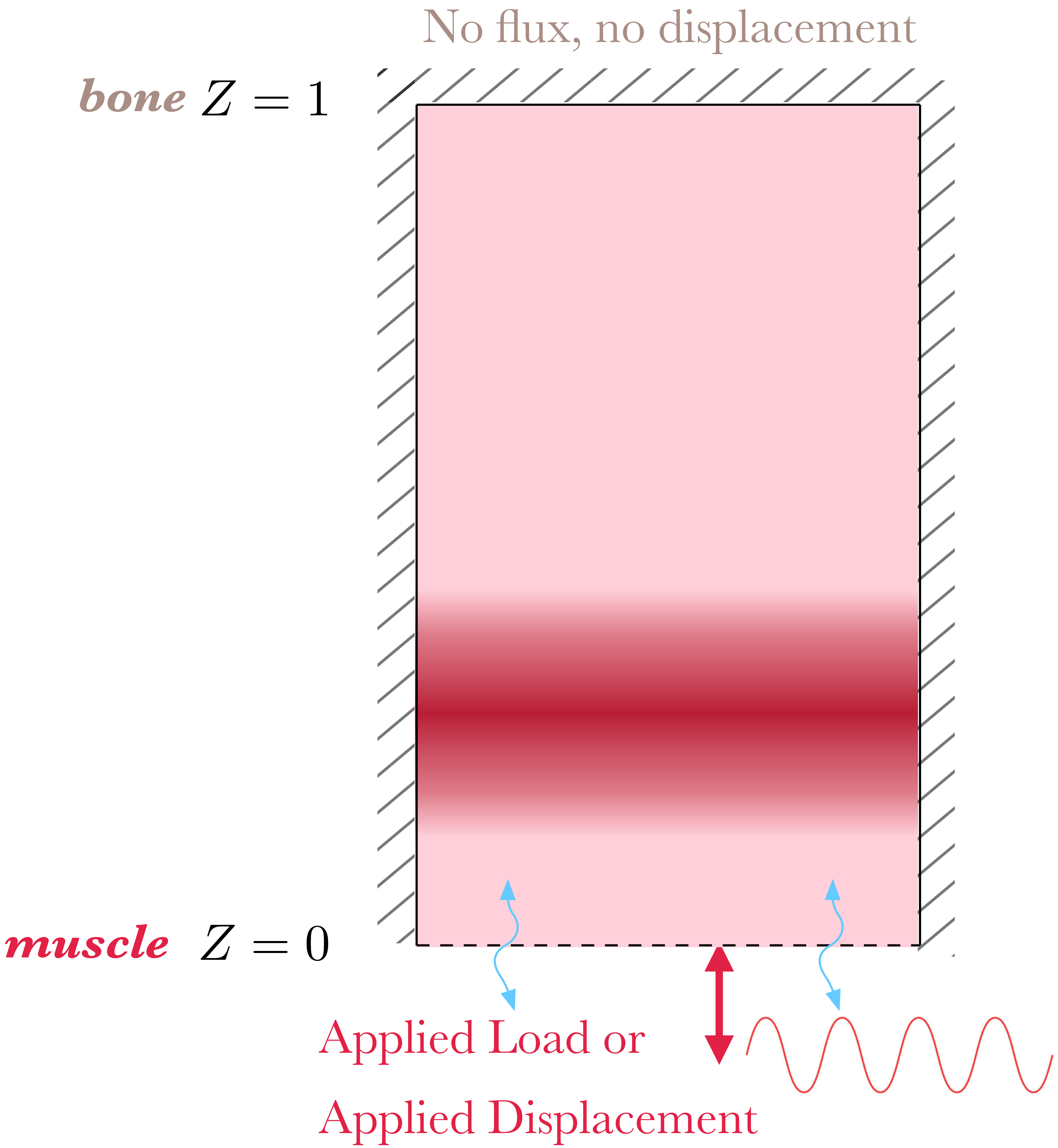
Non-linear advection diffusion equation for porosity

Heterogeneity

$$f(Z) = 1 - d\exp\left(-\frac{(Z-l)^2}{2c^2}\right)$$



$$M_d(Z) = f(Z)$$

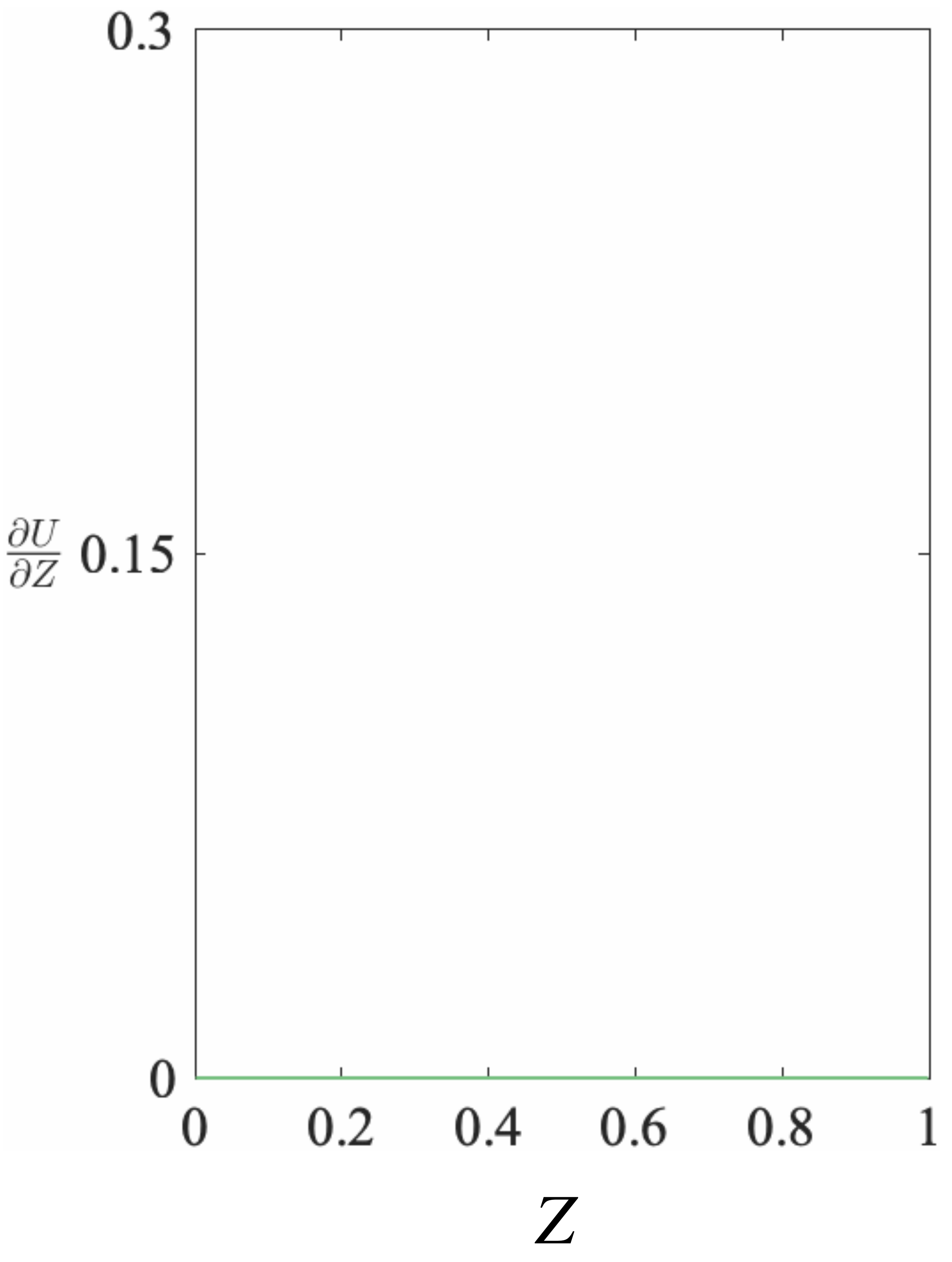


M Stiffness

undamaged

damaged

APPLIED LOAD - STRAIN / POROSITY RESPONSE (medium frequency)

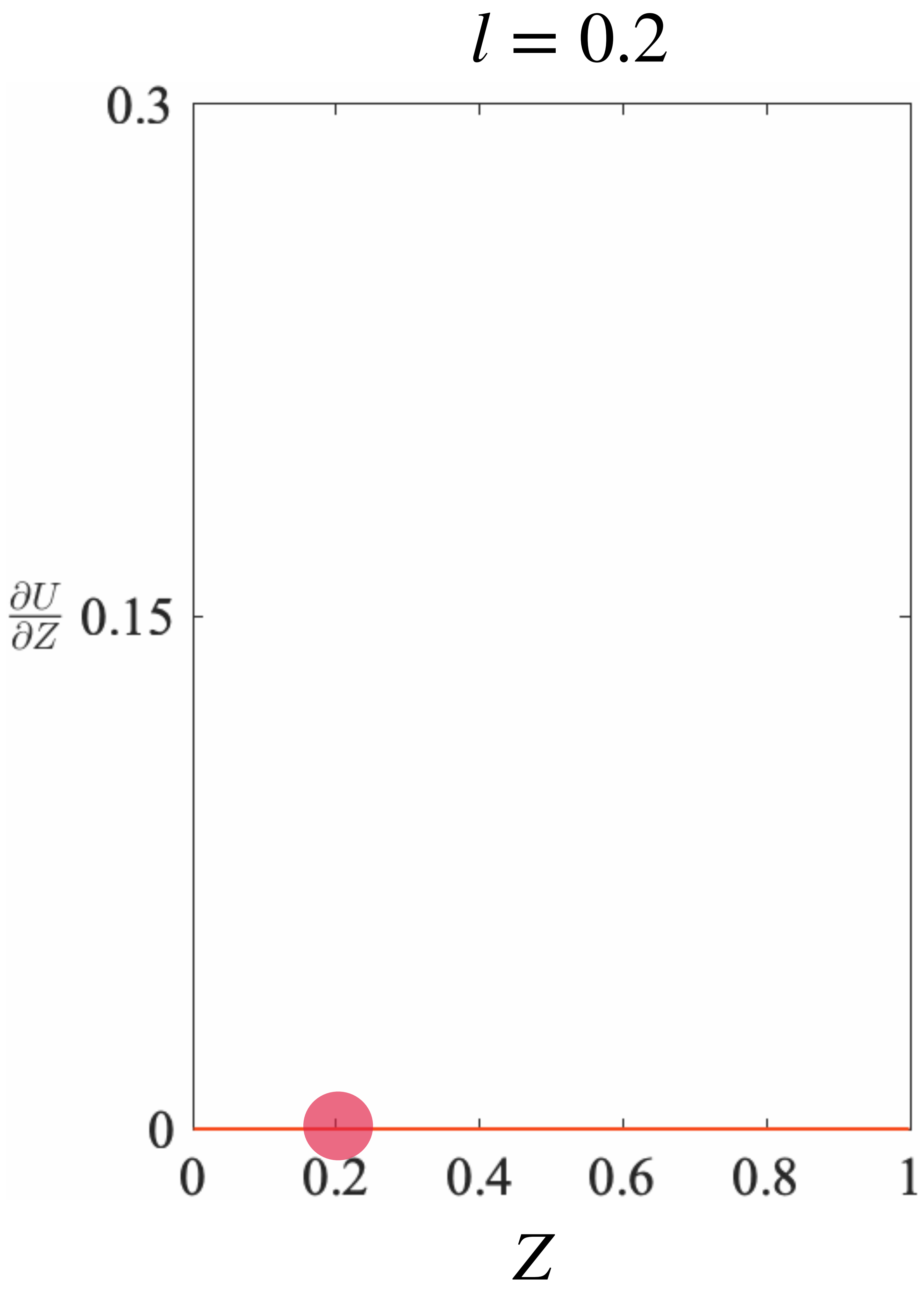


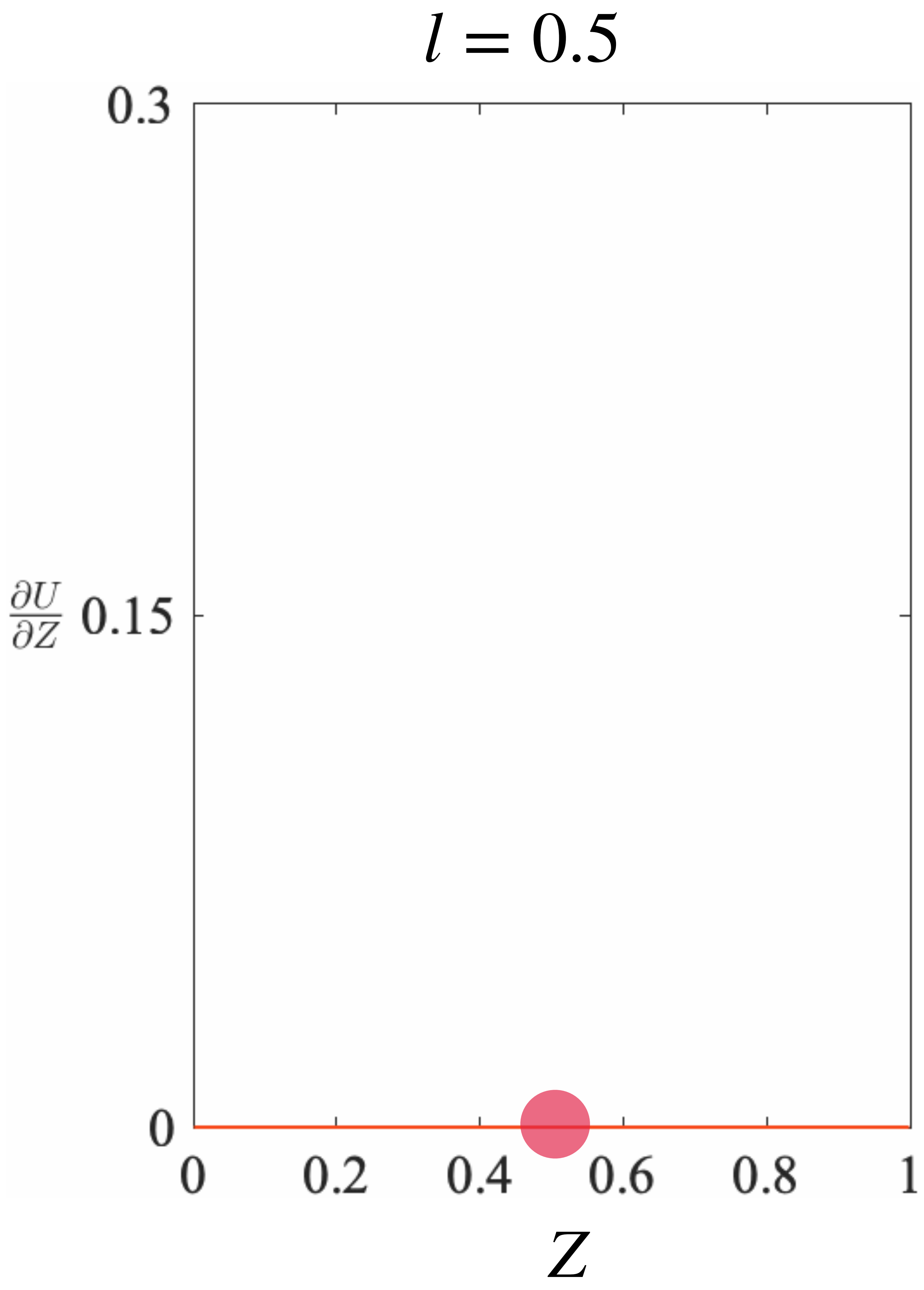
$$\frac{\partial U}{\partial Z} = \Phi - \Phi_0$$ Strain

- Fluid enters when pulled, exits when let go
- Positive feedback loop from porosity-dependent permeability
- Strain / porosity maintained as $Z \to 1$

APPLIED LOAD - STRAIN / POROSITY RESPONSE
(medium frequency)

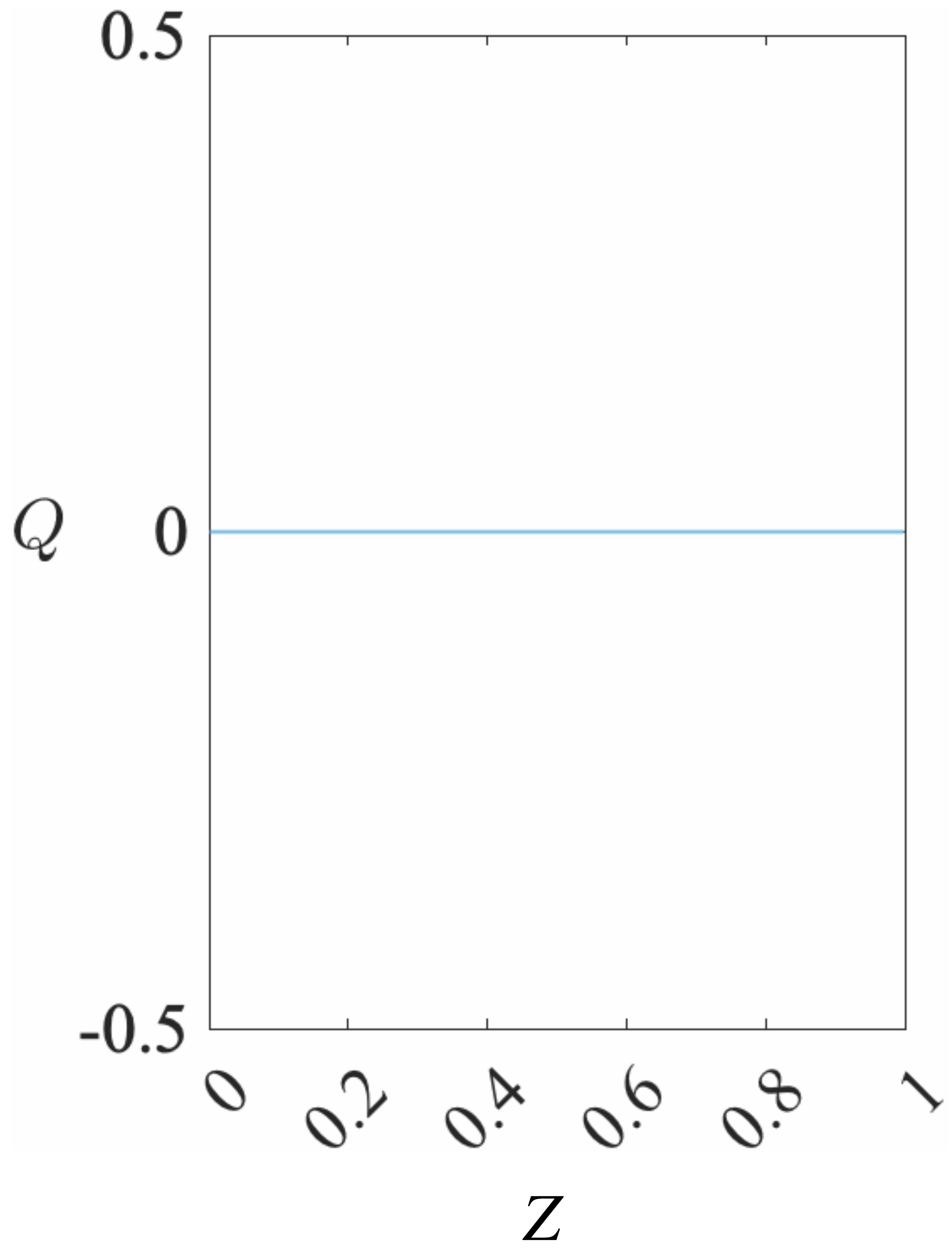
DAMAGED STIFFNESS





Strain increases around point of damage

APPLIED DISPLACEMENT - FLUX RESPONSE (medium frequency)

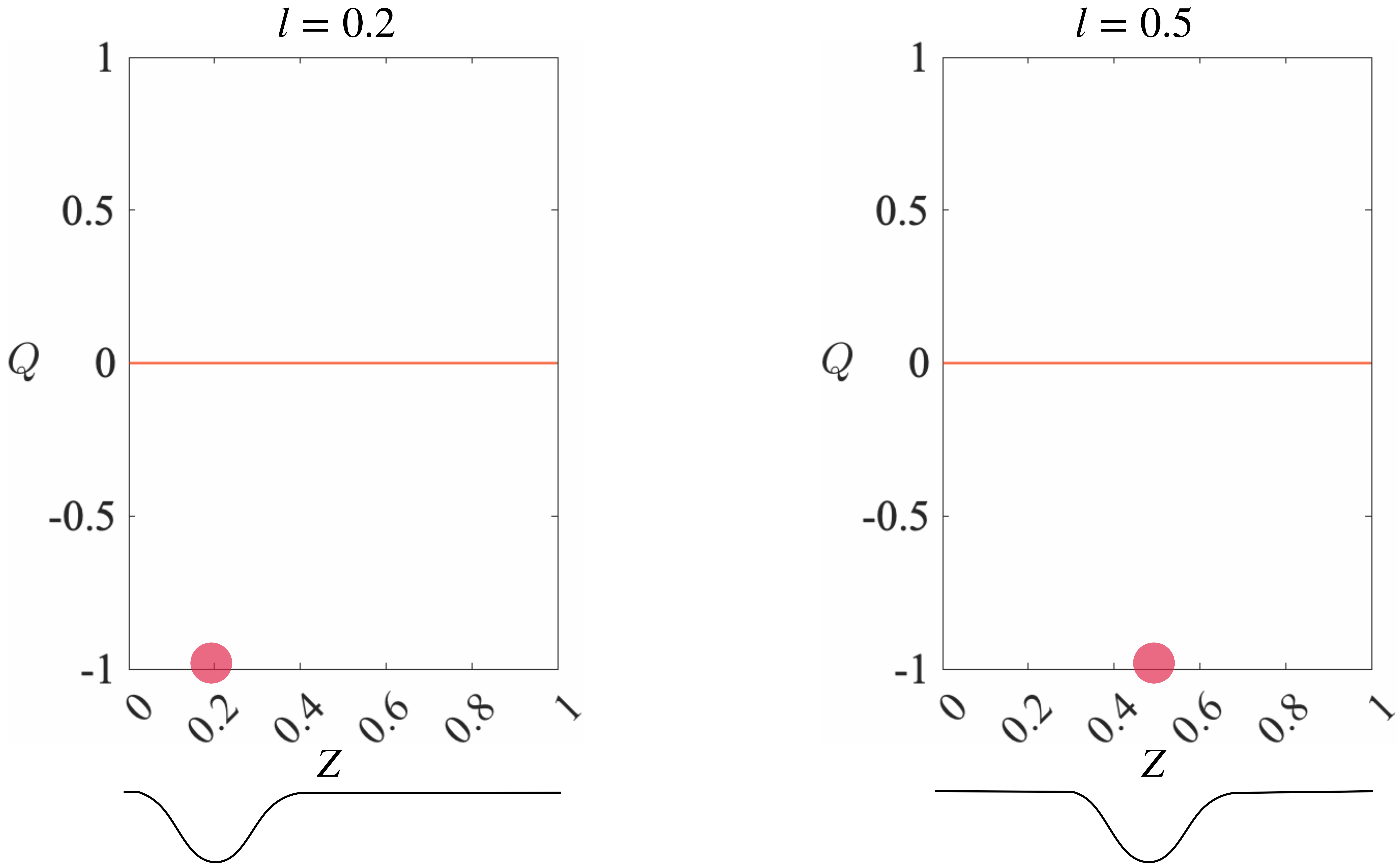


Q Relative Flow

- Fluid enters when pulled, exits when pushed back
- No flux at $Z = 0$

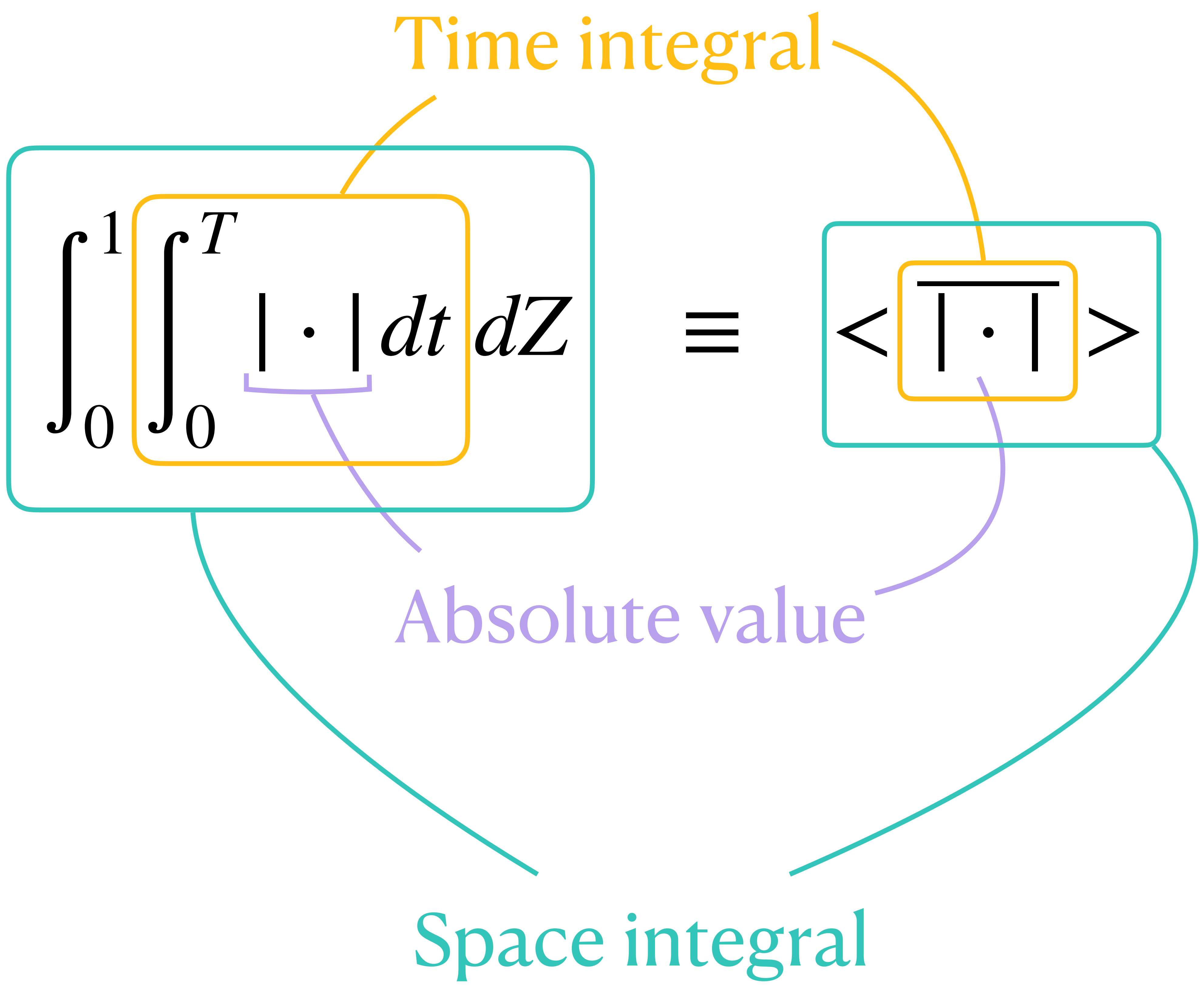
APPLIED DISPLACEMENT - FLUX RESPONSE
(medium frequency)

DAMAGED STIFFNESS

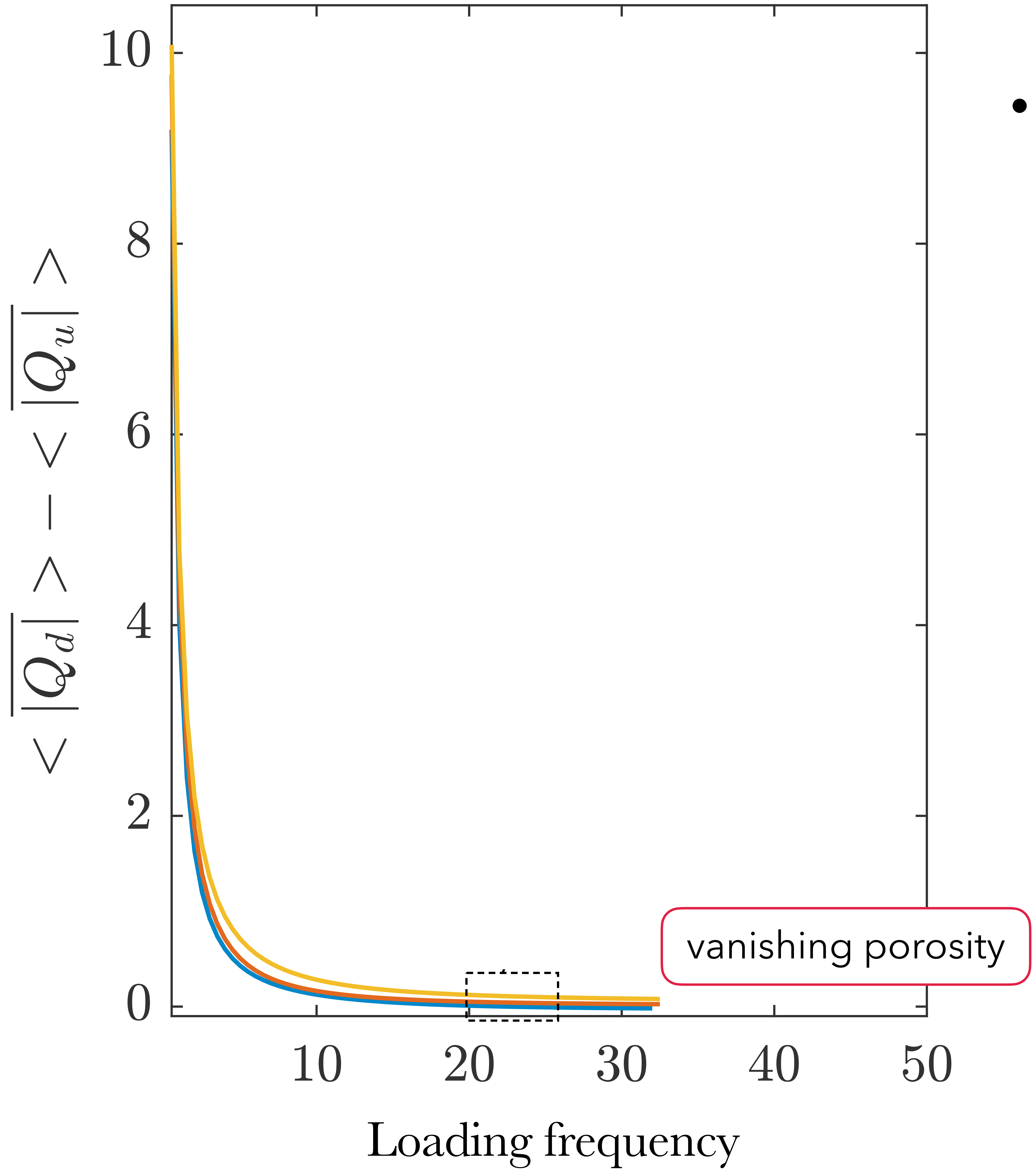


Generated flux into damaged region due to stress gradient (mostly)

How can we characterise how the response changes with frequency, damage magnitude and location?

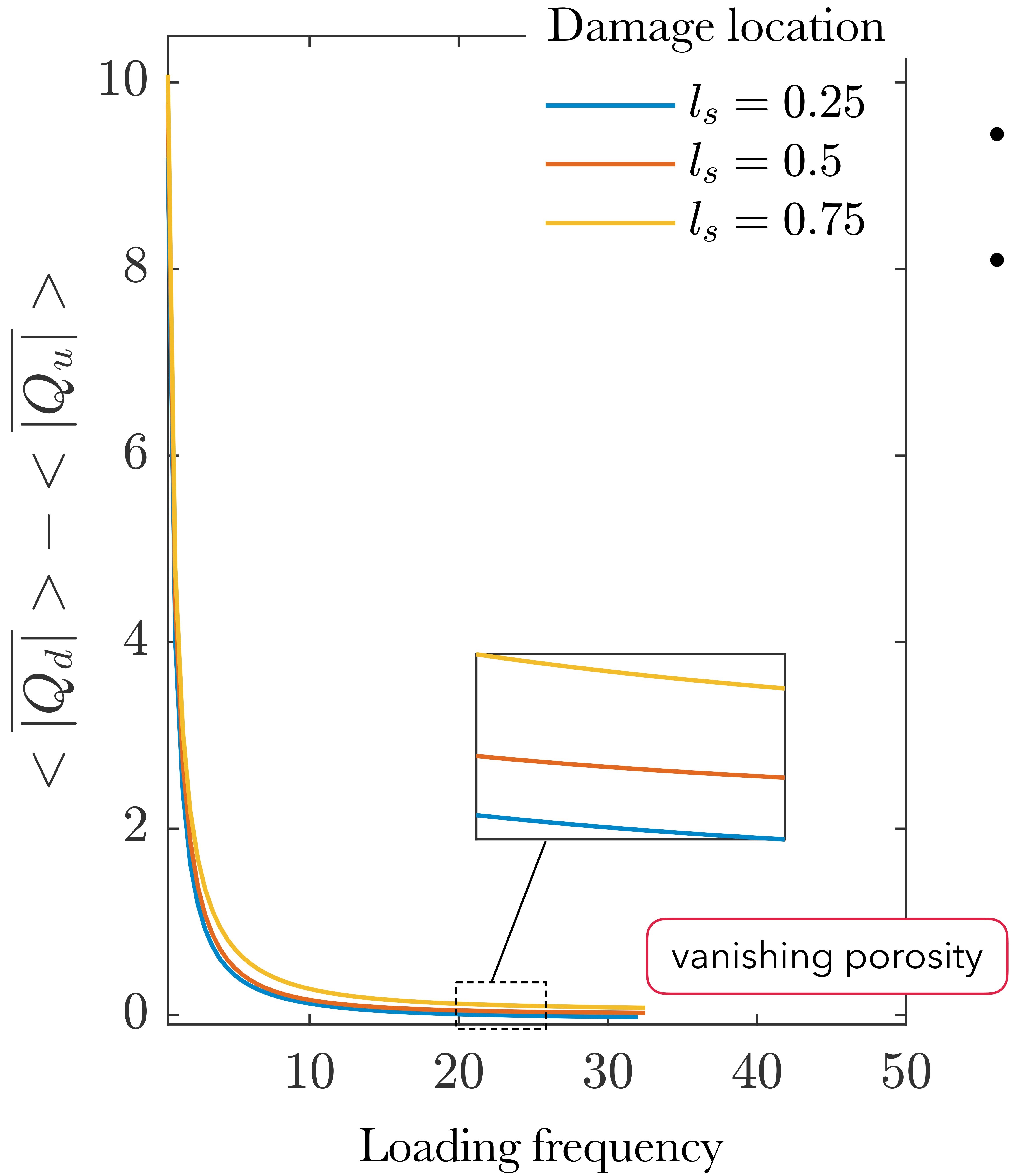


APPLIED DISPLACEMENT, varying loading frequency

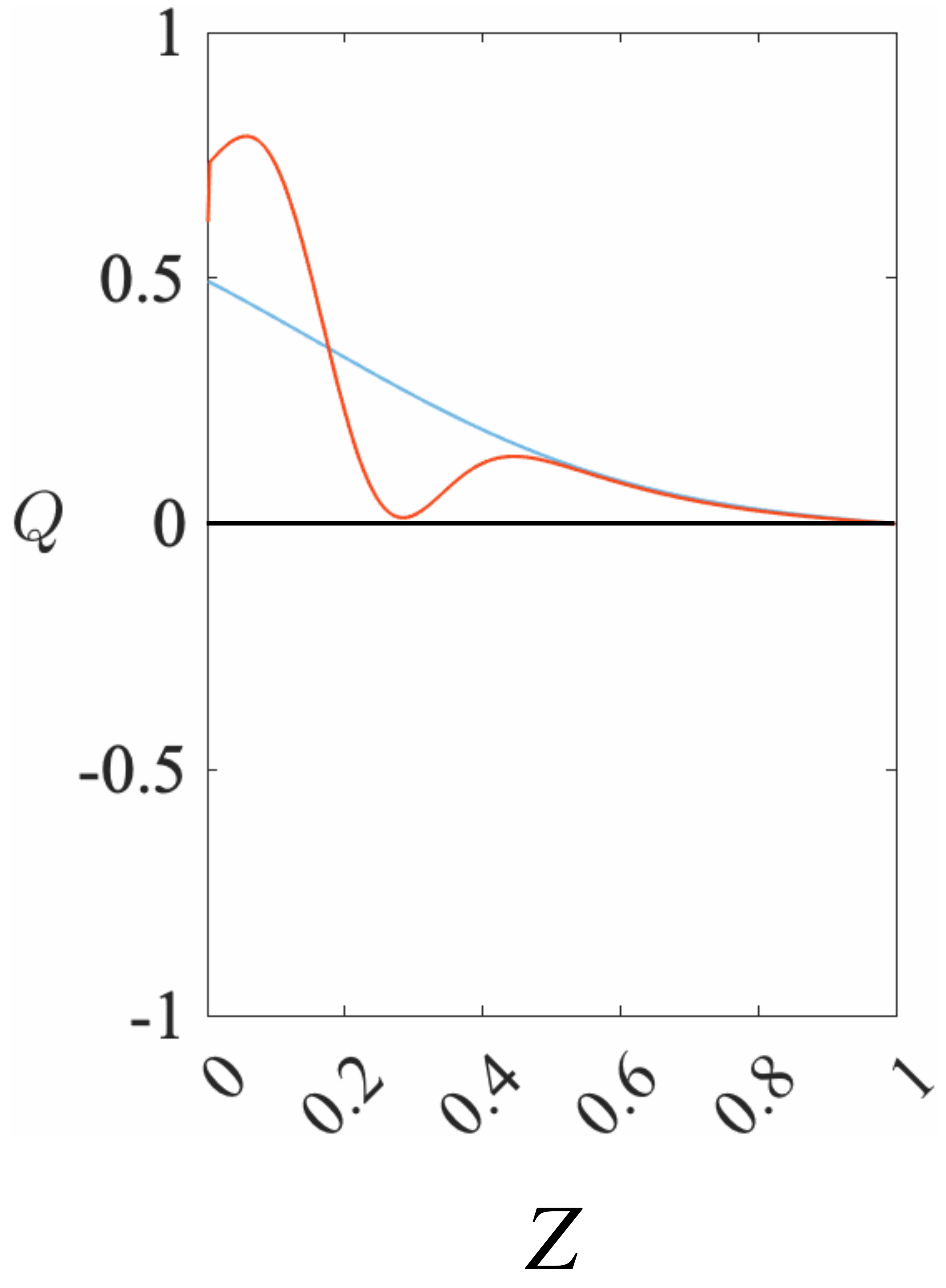


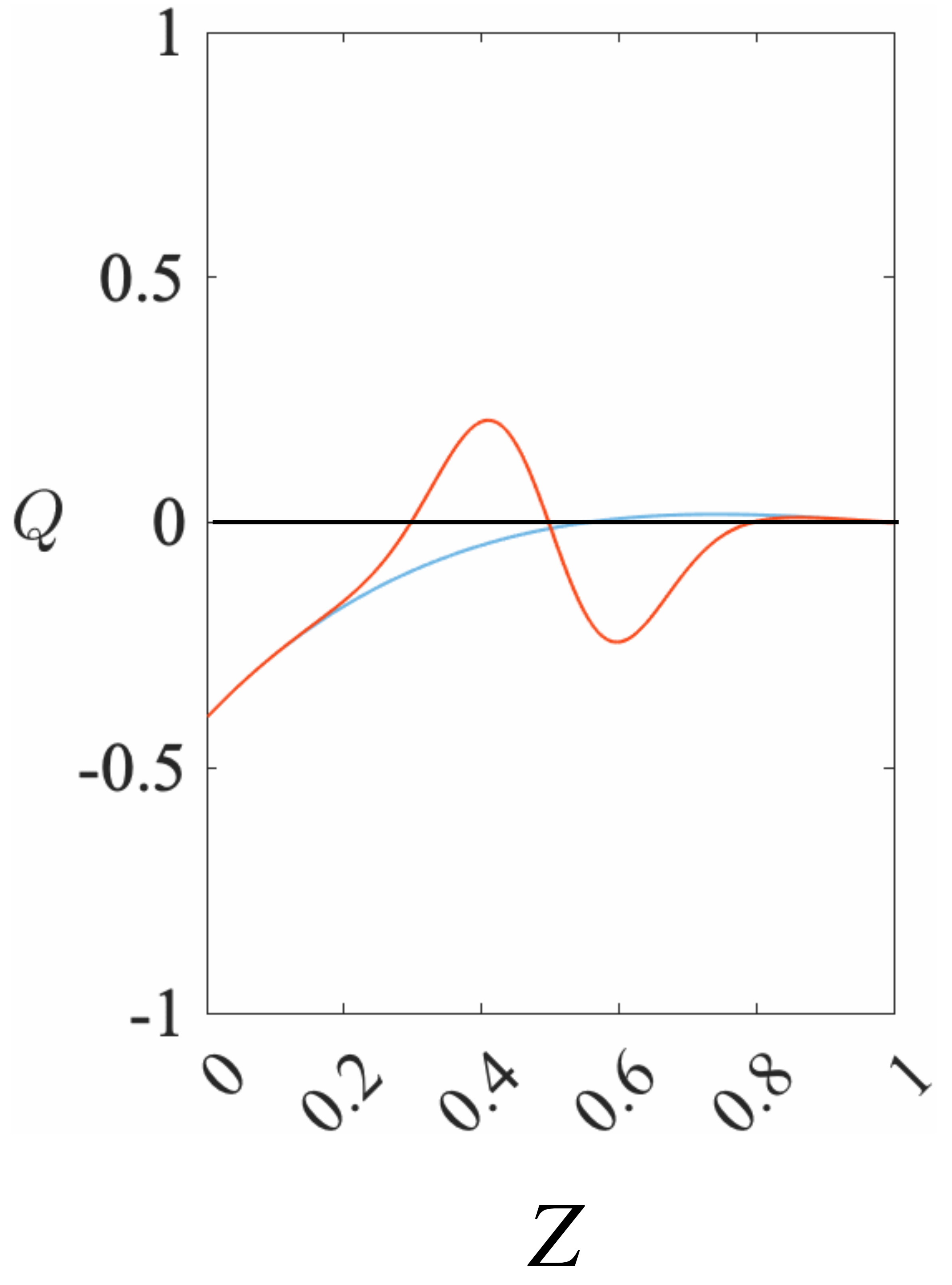
- Slower loading results in greater flux

APPLIED DISPLACEMENT, varying loading frequency & location



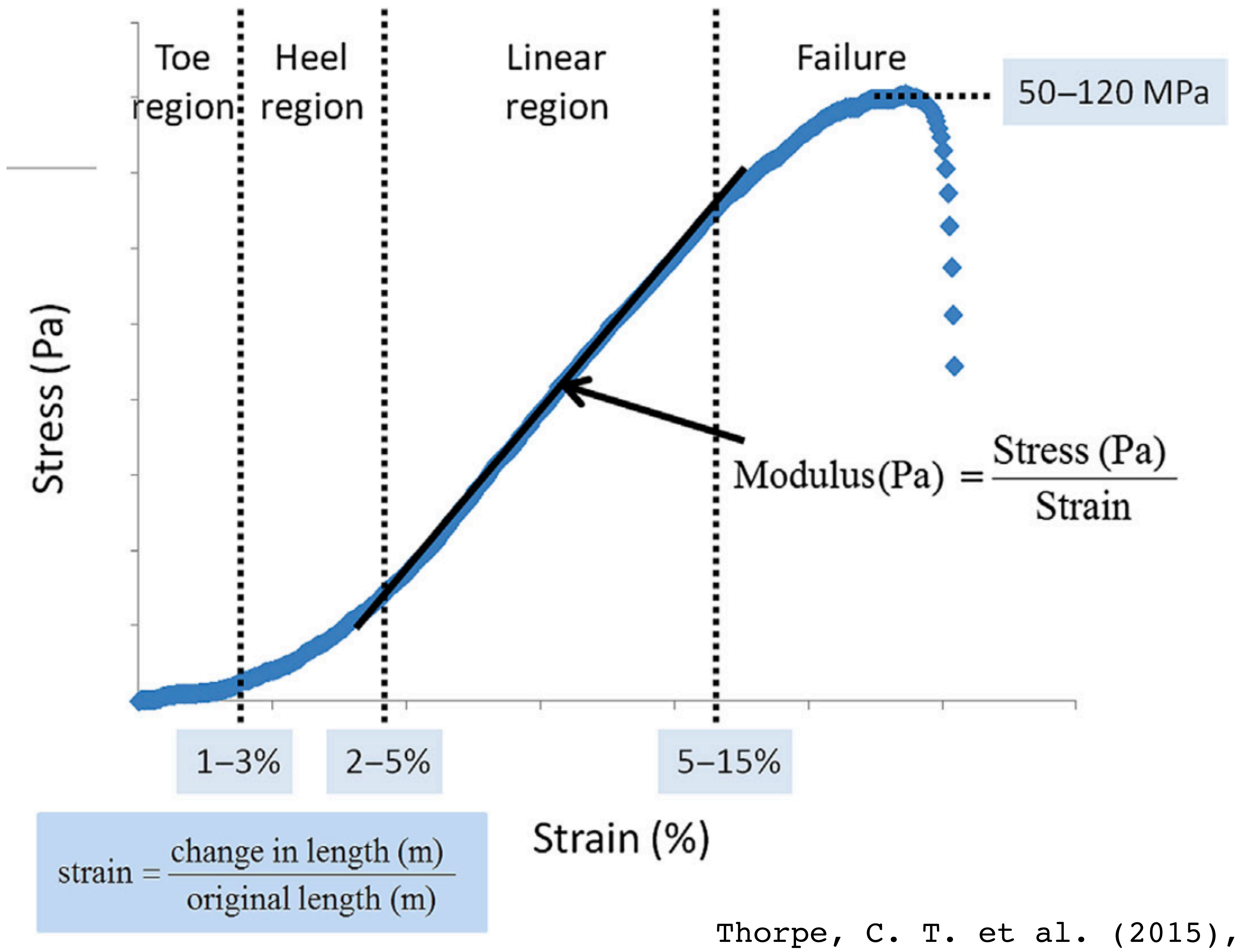
- Slower loading results in greater flux
- Damage further away from boundary results in greater flux





Broader context and future directions

- Many parameters and variables to play with: which ones are important?
- Informed by context of problem
- Aim: build a more accurate model for tendon



Thorpe, C. T. et al. (2015), *Tendon regeneration*

Tendon stress-strain curve

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Thank you for listening !

Any questions?

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