On semi-discrete finite-element schemes for energy-dissipating solutions to stochastic thin-film equations

Recently, we have shown that martingale solutions to stochastic thin-film equations

$$du + (u^n u_{xxx})_x \, dt - (C_{Strat} + S) \left(u^{n-2} u_x\right)_x dt = \left(u^{n/2} dW_Q(t)\right)_x, \tag{1}$$

which dissipate the surface-tension energy $\frac{1}{2}\int_{\mathcal{O}} |u_x|^2 dx$, have the property of finite speed of propagation. Here, $n \in (2,3)$, the term $-C_{Strat}\left(u^{n-2}u_x\right)_x dt$ is the Stratonovich correction term, and S is a positive parameter. Such energy-dissipating solutions to compactly supported initial data may be constructed as singular limit $\varepsilon \to 0$ of the SPDE

$$du + (u^n (u_{xx} - \varepsilon F'(u))_x)_x \, dt - (C_{Strat} + S) \left(u^{n-2} u_x\right)_x dt = \left(u^{n/2} dW_Q(t)\right)_x. \tag{2}$$

Here, $F(u) := \frac{1}{p} u^{-p}$ is a so-called effective interface potential which appears in certain wetting models to describe attractive interactions between liquid film and solid substrate. In this talk, we focus on equation (2) and we present a finite-element scheme which preserves the energetic structure of the equation and which is used to prove the existence of almost surely strictly positive solutions to (2).

The main analytical novelty of this approach is a discretization method which shows nonnegativity for a finite-element counterpart of the integral $\int_{\mathcal{O}} \left(u^{n-2}u_x\right)_x u_{xx} dx$ under periodic boundary conditions in the parameter regime $n \in (2,3)$. A control of such terms is essential to control Itô-correction terms in decay estimates for the surface-tension energy. This way, it is a key ingredient to establish the aforementioned singular limit $\varepsilon \to 0$ for the existence proof of energy dissipating solutions to (1) under compactly supported initial data.

This talk is based on joint works with Lorenz Klein (Erlangen).